Rotate The Region Bounded By Y=2x+1,x=4 And Y=3 About The Line X=-4 3.Rotate The Region Bounded By X=y-6y+10andx=5about

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Introduction

Understanding how to find the volume generated by rotating a region around a specific line is a fundamental concept in calculus, especially in the context of volumes of revolution. These problems often involve determining the bounds of the region, setting up the appropriate integral, and choosing the correct method of rotation—either the disk/washer method or the shell method. In this article, we will explore two specific problems:

- Rotating the region bounded by the curves y=2x+1, x=4, and y=3 about the line x=-4.
- Rotating the region bounded by the curve x=y-6y+10 and x=5 about an axis.

By analyzing these problems step-by-step, providing detailed explanations, and discussing the relevant calculus techniques, this article aims to enhance your understanding of volumes of revolution and their applications.

Understanding the First Problem: Rotation of the Region Bounded by Y=2x+1, x=4, and Y=3 About the Line X=-4

1.1 The Geometric Region

The initial step in solving these problems is to visualize and understand the region that is being rotated. The region is bounded by:

- The line y=2x+1
- The vertical line x=4
- The horizontal line y=3

Plotting these boundaries:

- The line $y=2x+1$ is a straight line with a slope of 2 and y-intercept at 1.
- The vertical boundary $x=4$ is a vertical line.
- The horizontal boundary $y=3$ is a horizontal line.

The intersection points:

- Find where $y=2x+1$ intersects $y=3$:

$3 = 2x + 1 \rightarrow 2x = 2 \rightarrow x=1$

So, the point of intersection: (1,3)

- The region is therefore enclosed between $x=1$ and $x=4$, from $y=2x+1$ up to $y=3$.

1.2 Setting Up the Integral

The goal is to find the volume when this region is revolved about the line $x=-4$.

Method choice:
Because the axis of rotation is vertical ($x=-4$), and the region extends horizontally between $x=1$ and $x=4$, the shell method is generally more straightforward.

Shell method overview:
- Consider vertical strips at position x, with thickness dx.
- When rotated about $x=-4$, each strip forms a cylindrical shell.

Shell parameters:

- Radius: Distance from the shell to the line $x=-4$:
$r(x) = x - (-4) = x + 4$

- Height: The vertical extent of the shell is from $y=2x+1$ up to $y=3$:

$h(x) = 3 - (2x + 1) = 2 - 2x$

Note: The height must be positive, so x must satisfy $2 - 2x \geq 0 \Rightarrow x \leq 1$. But since the region extends from $x=1$ to $x=4$, and the line $y=2x+1$ is increasing, the height is:

- For x in [1, 1], height is zero at $x=1$.
- For x in (1, 4], height is $3 - (2x + 1)$.

But because at $x=1$, $y=3$, and the lower boundary is $y=2x+1$, which is also 3 at $x=1$, the region is bounded between these x-values.

Integral for volume:

$$
V = 2\pi \int_{x=1}^{x=4} r(x) \cdot h(x) \, dx
$$

$$
V = 2\pi \int_{1}^{4} (x + 4) \cdot [3 - (2x + 1)] \, dx
$$

Simplify the integrand:

$$
3 - (2x + 1) = 2 - 2x
$$

Therefore,

$$
V = 2\pi \int_{1}^{4} (x + 4)(2 - 2x) \, dx
$$

Solving the Integral

2.1 Simplify the Expression

Expand the integrand:

$$
(x + 4)(2 - 2x) = x \cdot 2 - x \cdot 2x + 4 \cdot 2 - 4 \cdot 2x
$$

$$
= 2x - 2x^2 + 8 - 8x
$$

Combine like terms:

$$
(-2x^2) + (2x - 8x) + 8 = -2x^2 - 6x + 8
$$

2.2 Compute the Integral

\[
V = 2\pi \int_{1}^{4} (-2x^2 - 6x + 8) \, dx
\]

Calculate the indefinite integral:

\[
\int (-2x^2 - 6x + 8) \, dx = -\frac{2}{3}x^3 - 3x^2 + 8x + C
\]

Evaluate from 1 to 4:

\[
V = 2\pi \left[-\frac{2}{3}(4)^3 - 3(4)^2 + 8(4) - \left(-\frac{2}{3}(1)^3 - 3(1)^2 + 8(1) \right) \right]
\]

Compute step-by-step:

- For x=4:

\[
-\frac{2}{3} \times 64 - 3 \times 16 + 8 \times 4 = -\frac{128}{3} - 48 + 32
\]

- For x=1:

\[
-\frac{2}{3} \times 1 - 3 \times 1 + 8 \times 1 = -\frac{2}{3} - 3 + 8
\]

Now, subtract:

\[
V = 2\pi \left[\left(-\frac{128}{3} - 48 + 32 \right) - \left(-\frac{2}{3} - 3 + 8 \right) \right]
\]

Simplify each:

- First bracket:

\[

-\frac{128}{3} - 48 + 32 = -\frac{128}{3} - 16
\]

- Second bracket:

\[
-\frac{2}{3} - 3 + 8 = -\frac{2}{3} + 5
\]

Subtracting:

\[
\left(-\frac{128}{3} - 16 \right) - \left(-\frac{2}{3} + 5 \right) = -\frac{128}{3} - 16 + \frac{2}{3} - 5
\]

Combine fractions:

\[
-\frac{128}{3} + \frac{2}{3} = -\frac{126}{3} = -42
\]

Combine constants:

\[
-16 - 5 = -21
\]

Total:

\[
-42 - 21 = -63
\]

Finally, multiply by \(2\pi \):

\[
V = 2\pi \times (-63) = -126\pi
\]

Since volume cannot be negative, we take the absolute value:

\[
\boxed{V = 126\pi}

$$

Thus, the volume of the solid formed by revolving the specified region about x=-4 is 126π cubic units.

Understanding the Second Problem: Rotation of the Region Bounded by X=y-6y+10 and x=5 About an Axis

3.1 Clarification of the Region

The boundary described as "X=y-6y+10" appears to be a typo or a misinterpretation. Likely, the intended boundary is:

$$
x = y - 6y + 10
$$

which simplifies to:

$$
x = -5y + 10
$$

Assuming this is correct, the region is bounded by:

- The line $x = -5y + 10$
- The vertical line $x=5$

The region is in the xy-plane, with the boundary x = -5y + 10 and x=5.

3.2 Visualizing the Region

Plotting:

- The line $x = -5y + 10$ is a straight line with slope $-\frac{1}{5}$.
- The vertical boundary $x=5$ is a line.

Find where these two lines intersect:

$$
-5y + 10 = 5 \Rightarrow -5y = -5 \Rightarrow y=1

\]

At $y=1$, $x=5$.

Similarly, determine the

Frequently Asked Questions

How do you set up the integral to find the volume when rotating the region bounded by $y=2x+1$, $x=4$, and $y=3$ about the line $x=-4$?

First, rewrite the equations in terms of y to express x as a function of y: $x = (y - 1)/2$. Determine the y-values where the region is bounded, which are from $y=3$ up to y corresponding to $x=4$ (i.e., $y=24+1=9$). The radius for each shell is the distance from x to the line $x=-4$, which is $x + 4$. The integral is set up as ∫ from $y=3$ to $y=9$ of $2\pi(\text{radius})(\text{height})\, dy$, where height is dx/dy or derived from $x=y-1/2$.

What is the method to find the volume of the region bounded by $y=2x+1$, $x=4$, and $y=3$ when rotated about $x=-4$?

Use the shell method by integrating with respect to y. For each y, the shell radius is $(x + 4)$, and the shell height is the difference in x-values at that y (from $x=(y-1)/2$ to $x=4$). The volume is then the integral of 2π times radius times height over the y-interval from $y=3$ to $y=9$.

How do you determine the limits of integration for rotating the region bounded by $y=2x+1$, $x=4$, and $y=3$ about $x=-4$?

Find the y-values where the boundaries intersect: at $x=4$, $y=24+1=9$; at $y=3$, the lower boundary. The y-limits are from $y=3$ up to $y=9$, corresponding to the region between these y-values.

How do you interpret the rotation of the region bounded by $y=2x+1$, $x=4$, and $y=3$ about $x=-4$ in terms of volume calculation?

Rotating this region about the line $x=-4$ creates a solid of revolution resembling a shell or a washer, depending on the method. Using the shell method, each shell has radius $(x+4)$ and height based on the x-boundaries, integrated over the y-interval.

What is the process to find the volume of the region bounded by $x=y-6$,

y+10, and x=5 about the line x=-4?

First, find the points of intersection to determine the limits of integration in y. Then, express x in terms of y from x=y-6, and note that x=5 is a vertical boundary. Use the shell method: for each y, the shell radius is (x+4), and the height is the difference in x-values (from x=y-6 to x=5). Integrate over the y-limits to find the volume.

How do you set up the integral for rotating the region bounded by x=y-6, y+10, and x=5 about x=-4?

Express x=y-6, which is the left boundary, and x=5, the right boundary. For each y, the shell radius is (x+4), and the shell height is (from x=y-6 to x=5). The y-limits are from the y-value where x=y-6 intersects y+10 (solve for y). The volume is ∫ 2π(radius)(height) dy over these limits.

What are the key steps to compute the volume of the region bounded by x=y-6 and y+10 when rotated about x=-4?

Identify the intersection points to determine y-limits, express x in terms of y, and set up the shell method integral with radius (x+4) and height (difference between x=5 and x=y-6). Integrate over y to find the total volume.

How do you verify the bounds and functions when setting up these volume integrals?

Graph the region to visualize boundaries and intersections, solve equations to find intersection points, and confirm the limits of integration correspond to the region's y- or x-values. Double-check the expressions for radius and height to ensure they accurately represent the shell dimensions.

What are common mistakes to avoid when calculating volumes of rotated regions in these problems?

Common mistakes include mixing up the limits of integration, miscalculating the radius or height of shells, neglecting to convert equations properly when changing variables, and forgetting to account for the correct axis of rotation. Always verify the region and boundaries carefully before integrating.

Additional Resources

Rotations of Bounded Regions: An Expert Analysis of Geometric Transformations and Volume Calculations

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Introduction

The study of rotating bounded regions about specified axes is a fundamental topic in calculus, particularly in the context of finding volumes of solids of revolution. These problems, often encountered in advanced calculus courses, combine geometric intuition with analytical techniques such as the disk/washer method and the shell method. Today, we delve deeply into two specific problems involving the rotation of regions bounded by various curves around given lines. By dissecting each problem step-by-step, we aim to provide an in-depth understanding suitable for students, educators, and enthusiasts seeking mastery in this area.

Problem 1: Rotating the Region Bounded by $y = 2x + 1$, $x = 4$, and $y = 3$ About the Line $x = -4$

Understanding the Region

This problem involves a triangular-like region defined by three boundaries:

- The line $y = 2x + 1$, which is a straight line with slope 2 and y-intercept at (0, 1).
- The vertical line $x = 4$, which is a boundary on the right.
- The horizontal line $y = 3$, which caps the region from above.

Visualizing the region:

- The line $y = 2x + 1$ crosses the y-axis at (0, 1).
- When $y = 3$, the corresponding x on the line is $x = (3 - 1)/2 = 1$.
- The vertical line $x = 4$ extends from the intersection point with $y = 2x + 1$ at $y = 2(4) + 1 = 9$ down to $y = 3$.

Intersections:

- The intersection of $y=2x+1$ with $y=3$ occurs at $x = 1$.

Therefore, the bounded region is between:

- $x=1$ (on $y=2x+1$ at $y=3$)
- $x=4$
- The lines $y=3$ and $y=2x+1$

This creates a trapezoidal region extending from $x=1$ to $x=4$, between $y=3$ and $y=2x+1$.

Step 1: Setting Up the Integral for Rotation

The goal is to find the volume generated by rotating this region about the line $x = -4$.

Choosing the Method:

- Since the axis of rotation is vertical ($x = -4$), the shell method is most convenient because it involves integrating cylindrical shells parallel to the axis.

Shell Method Formula:

$$
V = 2 \pi \int_{x=a}^{x=b} (\text{radius}) \times (\text{height}) \, dx
$$

where:

- The radius is the distance from the shell to the axis $x = -4$,
- The height is the vertical extent of the region at each x.

Step 2: Determining the Limits and Components

- Limits: from $x=1$ to $x=4$.
- Radius: $r(x) = |x - (-4)| = x + 4$.
- Height: difference between the upper and lower boundaries for each x:

$$
\text{height} = y_{\text{top}} - y_{\text{bottom}} = 3 - (2x + 1) = 2 - 2x
$$

Note that for x between 1 and 2.5, the height is positive; beyond that, the region's height could become negative. But since $y = 2x + 1$ is increasing, and $y=3$ is constant, the vertical difference is valid between $x=1$ and $x=1.5$. Actually, at $x=1.5$, $y=2(1.5)+1=4$, which is above 3, so the region is between $y=2x+1$ and $y=3$ only where $y=2x+1 \leq 3$, i.e., $x \leq 1$.

Wait, this suggests a need to clarify the bounds:

- The region is bounded above by $y=3$ and below by $y=2x+1$, but only for y between these lines.

- The intersection point at $y=3$:

$$
2x + 1 = 3 \Rightarrow x=1
$$

- For $x<1$, the line $y=2x+1$ is below $y=3$, so the region extends from $x=0.5$ (say) up to $x=1$. But in our case, the problem states the region is between $y=2x+1$, $x=4$, and $y=3$. The key is that the region is between $x=1$ and $x=4$, with the upper boundary at $y=3$, and the lower boundary along $y=2x+1$.

- Therefore, for x in [1, 4], the vertical extent is from $y=2x+1$ (below) up to $y=3$ (above).

- Since the region between $y=2x+1$ and $y=3$ exists for x in [1, (3-1)/2=1], that indicates the region is only at $x=1$. But the problem states the region is bounded by these lines, so the region is between $y=2x+1$ and $y=3$, with x from 1 to where $2x+1=3$, i.e., $x=1$.

- But the problem mentions $x=4$, so perhaps the region is between $x=1$ and $x=4$, with the upper boundary $y=3$, and the lower boundary $y=2x+1$.

- To clarify, let's consider:

- For x between 1 and 4, the lower boundary is $y=2x+1$,
- The upper boundary is $y=3$.

- The region exists between these lines for x in [1, 1], which is a single point, and then from $x=1$ to $x=4$, the vertical extent is from $y=2x+1$ up to $y=3$.

- The intersection points:

$$
2x+1=3 \Rightarrow x=1
$$

- At $x=1$, both lines intersect at $y=3$.

- For $x<1$, $2x+1<3$, so the region is between those lines.

- For $x>1$, $2x+1>3$, so the lower boundary is above the upper boundary, which is impossible.

- Therefore, the region is between $x=0.5$ and $x=1$, with the lower boundary $y=2x+1$, and the upper boundary at $y=3$.

But since the problem explicitly states the region bounded by $y=2x+1$, $x=4$, and $y=3$, perhaps the intended region is the triangle between $x=1$ and $x=4$, bounded above by $y=3$ and below by \(y=2x+1 \).

Summary:

- The region is between:

- \(x=1 \) (where \(y=2x+1 \) meets \(y=3 \))
- \(x=4 \)
- \(y=2x+1 \) (below)
- \(y=3 \) (above)

- For \(x \) in [1, 4], the vertical extent:

\[
\text{height} = 3 - (2x + 1) = 2 - 2x
\]

which is positive only for \(x \leq 1 \). At \(x=1 \), height is zero. For \(x>1 \), \(2 - 2x < 0 \), implying the region is between \(y=2x+1 \) and \(y=3 \) only when \(2 - 2x > 0 \Rightarrow x<1 \).

This suggests an inconsistency. The likely correct description is:

- The region bounded by:

- \(y=2x+1 \),
- \(x=4 \),
- \(y=3 \),
- and the vertical boundary at \(x=1 \).

- The region is between \(x=1 \) and \(x=4 \), with the lower boundary \(y=2x+1 \), and the upper boundary \(y=3 \).

Rotate The Region Bounded By Y 2x 1 X 4 And Y 3 About The Line X 4 3 Rotate The Region Bounded By X Y 6y 10andx 5about

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