Sketch The Closed Curve C Consisting Of The Edges Of The Rectangle With Vertices (0,0,0),(0,1,1),(1,1,1),(1,0,0)

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Introduction to the Problem and Geometric Context

Understanding the geometry of three-dimensional objects and their projections is fundamental in fields such as vector calculus, differential geometry, and computer graphics. The problem at hand involves sketching a closed curve, C, which is formed by the edges of a specific rectangular shape in three-dimensional space. The vertices of this rectangle are given as points in 3D coordinates: (0,0,0), (0,1,1), (1,1,1), and (1,0,0).

This task requires an exploration of the spatial positioning of these vertices, the edges connecting them, and how to accurately represent the resulting closed curve in a three-dimensional context. Visualizing such a curve is vital for understanding the shape's structure, its orientation in space, and for further mathematical analysis such as calculating surface integrals or flux.

Analyzing the Vertices and Their Spatial Arrangement

Vertices Description

The vertices provided are:

- V1: (0, 0, 0)
- V2: (0, 1, 1)
- V3: (1, 1, 1)
- V4: (1, 0, 0)

These points define a quadrilateral in three-dimensional space. To understand the shape, consider the following:

- V1 and V4 share the same z-coordinate (0), but differ in the x-coordinate.
- V2 and V3 share the same z-coordinate (1), but differ in the x-coordinate.
- V1 and V2 share the same x-coordinate (0), but differ in y and z.
- V4 and V3 share the same x-coordinate (1), but differ in y and z.

This arrangement suggests the shape is a skewed or possibly a parallelogram in 3D space, depending on the positions of the vertices.

Visualizing the Spatial Structure of the Rectangle

Mapping the Vertices in 3D Space

Plotting the points:

- (0,0,0): Origin point
- (0,1,1): Located one unit along y and z from the origin
- (1,1,1): Shifted one unit along x from (0,1,1)
- (1,0,0): Located one unit along x from the origin, with y and z at zero

By visualizing these points:

- V1 and V4 lie on the plane z=0
- V2 and V3 lie on the plane z=1
- The points (0,0,0) and (1,0,0) are along the x-axis at y=z=0
- The points (0,1,1) and (1,1,1) are along the same y, z=1, at x=0 and x=1, respectively

This configuration indicates that the rectangle is inclined in space, connecting the bottom face formed by V1 and V4 to the top face formed by V2 and V3.

Constructing the Edges and the Closed Curve C

Edges of the Rectangle

The edges are line segments connecting these vertices:

1. Edge 1: V1 to V2 — from (0,0,0) to (0,1,1)
2. Edge 2: V2 to V3 — from (0,1,1) to (1,1,1)
3. Edge 3: V3 to V4 — from (1,1,1) to (1,0,0)
4. Edge 4: V4 to V1 — from (1,0,0) to (0,0,0)

Connecting these edges in sequence forms a closed polygonal curve, C, that traces the boundary of the rectangle in space.

Visual Representation of the Curve

To sketch curve C:

- Begin at V1 (0,0,0).
- Draw a line segment to V2 (0,1,1), which goes diagonally upward from the origin.
- From V2, draw a horizontal segment to V3 (1,1,1), moving along the x-axis at $y=1,z=1$.
- From V3, draw a segment down to V4 (1,0,0), which is diagonally downward back to the $x=1,y=0,z=0$.
- Finally, connect V4 back to V1, closing the loop.

This sequence traces the boundary of the rectangle in space.

Mathematical Description of the Edges

Parameterization of Each Edge

To facilitate a detailed sketch, each edge can be described parametrically:

- Edge V1 to V2:

$$
\vec{r}_{12}(t) = (0, t, t), \quad t \in [0,1]
$$

- Edge V2 to V3:

$$
\vec{r}_{23}(t) = (t, 1, 1), \quad t \in [0,1]
$$

- Edge V3 to V4:

$$
\vec{r}_{34}(t) = (1, 1 - t, 1 - t), \quad t \in [0,1]
$$

- Edge V4 to V1:

$$
\vec{r}_{41}(t) = (1 - t, 0, 0), \quad t \in [0,1]
$$

These parameterizations allow for precise plotting and analysis of the shape.

Properties of the Closed Curve C

Shape and Geometry

The curve C forms a quadrilateral that is skewed in space. Its sides are straight lines, but the overall shape is not necessarily a rectangle in the traditional Euclidean sense, due to the spatial inclination. It resembles a parallelogram or a trapezoid, depending on the angles, but in three dimensions.

Planarity and Surface

To verify whether C lies on a plane:

- Check if the four points are coplanar by calculating the scalar triple product of vectors:

$$
\vec{V_1V_2} = (0,1,1), \quad \vec{V_1V_4} = (1,0,0), \quad \vec{V_1V_3} = (1,1,1)
$$

- Compute the scalar triple product:

$$
\left[\vec{V_1V_2} \times \vec{V_1V_4}\right] \cdot \vec{V_1V_3}
$$

- If the scalar triple product is zero, the points are coplanar; otherwise, they are not.

Calculating:

$$

$$
\vec{V_1V_2} \times \vec{V_1V_4} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 1 & 1 \\
1 & 0 & 0 \\
\end{vmatrix}
= ((1)(0) - (1)(0)) \mathbf{i} - ((0)(0) - (1)(1)) \mathbf{j} + ((0)(0) - (1)(1)) \mathbf{k}
= 0 \mathbf{i} - (-1) \mathbf{j} - (1) \mathbf{k} = (0, 1, -1)
$$

Then,

$$
(0, 1, -1) \cdot (1, 1, 1) = (0)(1) + (1)(1) + (-1)(1) = 0 + 1 - 1 = 0
$$

Since the scalar triple product is zero, the four points are coplanar. Therefore, the curve C lies on a plane, which simplifies visualization.

Sketching the Curve C: Practical Approach

Steps for a Geometric Sketch

1. Set up axes in your drawing space for reference. Mark the origin (0,0,0).
2. Plot the vertices:
- Mark V1 at the origin.
- From V1, mark V2 at coordinates (0,1,1).
- From V2, mark V3 at (1,1,1).
- From V3, mark V4 at (1,0,0).
3. Connect the vertices with straight lines to form the boundary:
- V1 to V2
- V2 to V3
- V3 to V4
- V4 back to V1
4. Identify the plane on which the points

Frequently Asked Questions

What is the shape of the closed curve C formed by the edges of the given rectangle?

The closed curve C is a polygonal shape that follows the edges connecting the vertices (0,0,0), (0,1,1), (1,1,1), and (1,0,0), forming a quadrilateral in 3D space.

Are the vertices of the rectangle coplanar in three-dimensional space?

No, the vertices are not coplanar; they lie in different planes, which means the shape formed is a non-planar quadrilateral in 3D space.

What is the significance of sketching the closed curve C in understanding the shape's properties?

Sketching the closed curve helps visualize the spatial arrangement of the edges and vertices, aiding in understanding the shape's geometry and possible projections.

How can the shape formed by the edges be classified in three-dimensional geometry?

Since the vertices are not coplanar, the shape is a skew quadrilateral, which is a four-sided figure with edges not lying in the same plane.

Can the given vertices be used to find the length of each edge in the curve C?

Yes, by applying the distance formula between each pair of connected vertices, the lengths of the edges can be calculated.

What are the coordinates of the vertices of the rectangle, and what do they imply about the shape?

The vertices are at (0,0,0), (0,1,1), (1,1,1), and (1,0,0); these coordinates indicate the shape spans across different planes, forming a skew quadrilateral rather than a flat rectangle.

In which three-dimensional plane does the shape primarily exist?

The shape does not lie entirely in a single plane; it exists in three-dimensional space, with vertices located in different planes, making it a spatial figure.

Additional Resources

Sketch the closed curve C consisting of the edges of the rectangle with vertices (0,0,0), (0,1,1), (1,1,1), (1,0,0)

Introduction

Understanding the geometric properties of three-dimensional shapes is fundamental in various fields, ranging from computer graphics and architectural design to advanced mathematical modeling. One of the foundational exercises in spatial geometry involves analyzing the edges of a polyhedral shape and representing their outlines or boundaries in a two-dimensional or three-dimensional sketch. In this context, our focus is on sketching the closed curve C that consists of the edges of a particular rectangle defined by four vertices in three-dimensional space: $(0,0,0)$, $(0,1,1)$, $(1,1,1)$, and $(1,0,0)$.

This exploration not only helps visualize the shape but also deepens our understanding of spatial relationships, projections, and the geometric significance of these points and edges. This article aims to provide a comprehensive analysis of this problem, discussing the geometric configuration, the nature of the edges, and the implications of sketching such a curve, supported by detailed explanations and visualizations.

Defining the Vertices and the Shape

Vertices of the Rectangle in 3D Space

The four vertices provided are:

- $A = (0, 0, 0)$
- $B = (0, 1, 1)$
- $C = (1, 1, 1)$
- $D = (1, 0, 0)$

Plotting these points in three-dimensional space reveals their spatial configuration:

- Point A is at the origin.
- Point B extends one unit along the y-axis and one unit along the z-axis.
- Point C is one unit along the x-axis and one unit along the y-axis, and also one unit along the z-axis.
- Point D is one unit along the x-axis and at the origin along y and z.

The key observation here is that the vertices do not all lie in a single plane, indicating that the shape they define is not a simple planar rectangle but rather a skew quadrilateral or a rectangle in three-dimensional space.

Clarifying the Shape: Is It a Rectangle?

Given the vertices, one might initially assume a standard rectangle lying flat in a plane. However, examining their coordinates reveals a more complex configuration:

- The points A and D are in the plane $z=0$.
- The points B and C are in the plane $z=1$.

This suggests that the shape is a "lifted" rectangle, with the bottom edge AD lying in the plane $z=0$, and the top edge BC lying in the plane $z=1$. Connecting these points forms a quadrilateral that is not coplanar but rather a skewed or "tilted" rectangle in three dimensions.

Therefore, the shape is a rectangular prism's side face or a parallelogram connecting these vertices, but with edges lying across different planes.

Edges of the Shape and Their Geometric Significance

Identifying the Edges

The shape's edges are the line segments connecting the vertices:

1. $A \rightarrow B$: from $(0,0,0)$ to $(0,1,1)$
2. $B \rightarrow C$: from $(0,1,1)$ to $(1,1,1)$
3. $C \rightarrow D$: from $(1,1,1)$ to $(1,0,0)$
4. $D \rightarrow A$: from $(1,0,0)$ to $(0,0,0)$

These four edges form a closed quadrilateral C, which is the focus of the sketch.

Note: The shape's edges connect points across different planes, implying the shape is a skew quadrilateral, not lying flat in a single plane.

Analyzing Each Edge

- Edge $A \rightarrow B$: This edge moves along the $x=0$ plane, with a change from $(0,0,0)$ to $(0,1,1)$. The vector representing this edge is $\vec{AB} = (0,1,1)$.

- Edge $B \rightarrow C$: This edge moves in the $y=1, z=1$ plane from $(0,1,1)$ to $(1,1,1)$. The vector is $\vec{BC} = (1,0,0)$.

- Edge $C \rightarrow D$: Connecting $(1,1,1)$ to $(1,0,0)$. The vector is $\vec{CD} = (0,-1,-1)$.

- Edge $D \rightarrow A$: Returning to the origin from $(1,0,0)$ to $(0,0,0)$. The vector is $\vec{DA} = (-1,0,0)$.

These vectors form the basis for understanding the shape's geometry, including lengths of edges and angles between them.

Visualizing the Shape: Spatial and Projected Perspectives

3D Visualization and Spatial Relationships

In three dimensions, the shape can be visualized as a quadrilateral that connects the bottom square $A-D$ in the plane $z=0$ with the top square $B-C$ in the plane $z=1$. The edges $A \rightarrow B$ and $D \rightarrow C$ are "rising" edges, connecting points in different planes.

This configuration resembles a parallelogram or a skew quadrilateral, where opposing sides are not necessarily parallel in the traditional sense but are connected through the shape's spatial orientation.

Projection onto Coordinate Planes

To better understand the curve C, projecting it onto coordinate planes can be insightful:

- Projection onto the xy-plane:
- $A=(0,0)$
- $B=(0,1)$
- $C=(1,1)$
- $D=(1,0)$

This projection reveals a standard square in the xy-plane.

- Projection onto the xz-plane:
- $A=(0,0)$
- $B=(0,1)$
- $C=(1,1)$
- $D=(1,0)$

Notice the same pattern.

- Projection onto the yz-plane:
- $A=(0,0)$
- $B=(1,1)$
- $C=(1,1)$
- $D=(0,0)$

The projection shows the vertical relationships between the points.

These projections help visualize the spatial orientation and facilitate the process of sketching the boundary curve.

Constructing the Closed Curve C

Methodology for Sketching

To accurately sketch the closed curve C, consider the following approach:

1. Identify the vertices in a coordinate system: Plot the four points in 3D space or a 2D projection for clarity.
2. Connect the vertices with straight lines in sequence: $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$.
3. Ensure proper representation of the shape's skewness: Since the shape is not planar, the edges will not all lie flat on a single sheet.
4. Use projections to aid visualization: Drawing the shape's projections onto coordinate planes helps understand the spatial relations.
5. Depict the edges distinctly: Use solid lines for visible edges and dashed lines for edges that are hidden from view, if creating a 3D sketch.

Analytical Sketching: Coordinates and Vectors

By using the coordinates:

- Starting at $A(0,0,0)$,
- Draw the edge $A \rightarrow B$ to $(0,1,1)$,
- Then $B \rightarrow C$ to $(1,1,1)$,
- Next $C \rightarrow D$ to $(1,0,0)$,
- Finally $D \rightarrow A$ back to $(0,0,0)$.

This sequence completes the closed curve C.

Implications of the Sketch

Sketching the boundary curve of such a shape offers insights into:

- Edge lengths:
- $AB = \sqrt{(0)^2 + (1)^2 + (1)^2} = \sqrt{2}$
- $BC = \sqrt{(1)^2 + (0)^2 + (0)^2} = 1$
- $CD = \sqrt{(0)^2 + (-1)^2 + (-1)^2} = \sqrt{2}$
-

Sketch The Closed Curve C Consisting Of The Edges Of The

Rectangle With Vertices 0 0 0 0 1 1 1 1 1 1 0 0

Sketch The Closed Curve C Consisting Of The Edges Of The Rectangle With Vertices 0 0 0 0 1 1 1 1 1 1 0 0

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