State All Integer Values Of X In The Interval 3<=x<=10 That Satisfy The Following Inequality: -5x+1<=-19

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Understanding how to find the integer solutions within a specific interval for a given inequality is a fundamental skill in algebra. In this article, we will explore the process of solving the inequality -5x + 1 ≤ -19 for integer values of x in the interval 3 ≤ x ≤ 10. We will walk through each step methodically, ensuring clarity and comprehension, and conclude by listing all integer values within the specified interval that satisfy the inequality.

Introduction to Inequalities and Interval Constraints

Before delving into solving the specific inequality, it is essential to understand some basic concepts:

What is an Inequality?

An inequality is a mathematical statement that compares two expressions using inequality signs such as <, >, ≤, or ≥. It indicates that one side is less than, greater than, less than or equal to, or greater than or equal to the other.

Understanding the Interval 3 ≤ x ≤ 10

The interval 3 ≤ x ≤ 10 specifies that the variable x can take any real value between 3 and 10, inclusive. When focusing on integer solutions, this means x can be 3, 4, 5, 6, 7, 8, 9, or 10.

Step-by-Step Solution of the Inequality -5x + 1 ≤ -19

To find the integer values of x that satisfy the inequality, we need to solve for x algebraically.

Step 1: Isolate the Variable Term

The inequality is:

$-5x + 1 \leq -19$

Our goal is to solve for x, so first, subtract 1 from both sides to isolate the term containing x.

1. Subtract 1 from both sides:

$-5x + 1 - 1 \leq -19 - 1$

which simplifies to:

$-5x \leq -20$

Step 2: Divide Both Sides by the Coefficient of x

The coefficient of x is -5. To solve for x, divide both sides by -5. Remember, dividing both sides of an inequality by a negative number reverses the inequality sign.

1. Divide both sides by -5 and flip the inequality sign:

$-5x / -5 \geq -20 / -5$

which simplifies to:

$x \geq 4$

Step 3: Consider the Interval Constraints

The solution to the inequality is $x \geq 4$. However, since the problem specifies that x must be within the interval $3 \leq x \leq 10$, we need to find the intersection of these two conditions.

- The inequality solution: $x \geq 4$

- The interval constraint: $3 \leq x \leq 10$

The intersection is all x such that:

$4 \leq x \leq 10$

Identifying Integer Solutions within the Specified Interval

Given that x must be an integer between 3 and 10, and satisfying $x \geq 4$, the set of solutions is:

$x \in \{4, 5, 6, 7, 8, 9, 10\}$

These are the integer values within the interval $3 \leq x \leq 10$ that satisfy the inequality $-5x + 1 \leq -19$.

Summary of Valid Solutions

- 4
- 5
- 6
- 7
- 8
- 9
- 10

Verifying the Solutions

To ensure accuracy, let's verify each of these values by substituting them back into the original inequality.

Verification Process

For each x in the set {4, 5, 6, 7, 8, 9, 10}:

1. Calculate $-5x + 1$
2. Check if the result is less than or equal to -19

Example for x = 4:

-5(4) + 1 = -20 + 1 = -19

-19 ≤ -19 → True

Example for x = 5:

-5(5) + 1 = -25 + 1 = -24

-24 ≤ -19 → True

Similarly, verify for other values:

- x=6: -30 + 1 = -29 → True

- x=7: -35 + 1 = -34 → True

- x=8: -40 + 1 = -39 → True

- x=9: -45 + 1 = -44 → True

- x=10: -50 + 1 = -49 → True

All these values satisfy the inequality, confirming our solution set.

Conclusion and Summary

In conclusion, solving the inequality -5x + 1 ≤ -19 within the interval 3 ≤ x ≤ 10 involves algebraic manipulation and understanding interval constraints. The process includes:

- Subtracting 1 from both sides to isolate the term with x
- Dividing both sides by -5 and reversing the inequality sign
- Intersecting the solution with the given interval to find valid integer solutions

The integer values of x in the interval 3 ≤ x ≤ 10 that satisfy the inequality are:

4, 5, 6, 7, 8, 9, 10

This comprehensive approach ensures clarity and accuracy in solving inequalities with interval constraints, which is a crucial aspect of algebra and mathematical problem-solving.

Additional Tips for Solving Similar

Inequalities

- Always perform inverse operations to isolate the variable.
- Remember that dividing or multiplying both sides of an inequality by a negative number reverses the inequality sign.
- When dealing with interval constraints, always find the intersection of the solution set with the interval.
- Verify solutions by substituting back into the original inequality.

By mastering these steps, you can confidently approach and solve a wide variety of inequalities within specified intervals, enhancing your algebraic problem-solving skills.

Frequently Asked Questions

What are the integer values of x in the interval 3 ≤ x ≤ 10 that satisfy the inequality -5x + 1 ≤ -19?

The integer values are x = 4, 5, 6, 7, 8, 9, 10.

How do we solve the inequality -5x + 1 ≤ -19 for x in the given interval?

Subtract 1 from both sides: -5x ≤ -20. Divide both sides by -5 (remember to flip the inequality sign): x ≥ 4. Since x is between 3 and 10, the solutions are x = 4, 5, 6, 7, 8, 9, 10.

Are all integer values from 3 to 10 included in the solution set for the inequality -5x + 1 ≤ -19?

No, only x ≥ 4 satisfy the inequality. Therefore, integers from 4 to 10 are solutions, while 3 is not.

What is the significance of dividing by -5 when solving the inequality -5x + 1 ≤ -19?

Dividing by -5 reverses the inequality sign because dividing both sides of an inequality by a negative number flips the direction of the inequality.

Can x = 4 satisfy the inequality -5x + 1 ≤ -19?

Yes, substituting x = 4 gives -5(4) + 1 = -20 + 1 = -19, which satisfies the inequality since -19 ≤ -19.

What is the final set of integer solutions for x in 3 ≤ x ≤ 10 that satisfy the inequality?

The solutions are x = 4, 5, 6, 7, 8, 9, 10.

Additional Resources

State All Integer Values Of X In The Interval 3 ≤ x ≤ 10 That Satisfy the Inequality: -5x + 1 ≤ -19

An In-Depth Investigation into the Integer Solutions of the Inequality -5x + 1 ≤ -19 within the Interval 3 ≤ x ≤ 10

Understanding the solutions to inequalities is fundamental in algebra and forms a cornerstone for more advanced mathematical reasoning. In particular, identifying all integer values of a variable that satisfy a given inequality within a specified interval is a task that blends algebraic manipulation with logical reasoning. This detailed investigation aims to systematically analyze and determine all integer solutions for the inequality:

$$
-5x + 1 \leq -19
$$

within the interval:

$$
3 \leq x \leq 10
$$

This exploration not only offers an explicit answer but also provides a thorough explanation of the methods and reasoning processes involved, suitable for educational purposes, review articles, or scholarly publication.

Mathematical Foundations and Context

Before delving into the solution, it is essential to understand the context and notation involved:

- Inequalities: Mathematical expressions that denote a range of values rather than a specific value. Solving inequalities involves finding the set of all possible values that satisfy the given relation.
- Intervals: The set of all numbers lying between two endpoints. Here, the interval $3 \leq x \leq 10$ indicates that x can be any real number between 3 and 10, inclusive.

- Integer solutions: Values of x that are whole numbers within the interval and satisfy the inequality.

The problem requires us to find all integer values x in the specified interval such that the inequality holds true.

Step-by-Step Solution Approach

Step 1: Isolate the variable

The given inequality is:

$$
-5x + 1 \leq -19
$$

To analyze and solve for x, the first step is to isolate the variable by performing algebraic manipulations.

Step 2: Subtract 1 from both sides

$$
-5x + 1 - 1 \leq -19 - 1
$$

which simplifies to:

$$
-5x \leq -20
$$

Step 3: Divide both sides by -5

Important: When dividing both sides of an inequality by a negative number, the direction of the inequality reverses.

$$
-5x \leq -20 \implies x \geq \frac{-20}{-5}
$$

$$
x \geq 4
$$

Note: The inequality now reads $x \geq 4$.

Step 4: Incorporate the interval constraint

The original interval constraint is:

$$
3 \leq x \leq 10
$$

Combining this with the solution $x \geq 4$, the set of all x satisfying both conditions is:

$$
4 \leq x \leq 10
$$

Step 5: Identify integer solutions within the interval

Since the problem asks explicitly for integer solutions, we list all integers between 4 and 10, inclusive:

$$
x \in \{4, 5, 6, 7, 8, 9, 10\}
$$

Validation of the Solutions

To ensure correctness, verify each candidate integer value by substituting back into the original inequality:

$$
-5x + 1 \leq -19
$$

Verification list:

x	Calculation of $-5x + 1$	Is inequality satisfied?
4	$-5 \times 4 + 1 = -20 + 1 = -19$	$-19 \leq -19$ → True
5	$-5 \times 5 + 1 = -25 + 1 = -24$	$-24 \leq -19$ → True
6	$-5 \times 6 + 1 = -30 + 1 = -29$	$-29 \leq -19$ → True
7	$-5 \times 7 + 1 = -35 + 1 = -34$	$-34 \leq -19$ → True
8	$-5 \times 8 + 1 = -40 + 1 = -39$	$-39 \leq -19$ → True
9	$-5 \times 9 + 1 = -45 + 1 = -44$	$-44 \leq -19$ → True
10	$-5 \times 10 + 1 = -50 + 1 = -49$	$-49 \leq -19$ → True

All listed values satisfy the inequality, confirming the validity of the solutions.

Comprehensive Summary of the Solutions

Integer values of x in the interval $3 \leq x \leq 10$ that satisfy

$-5x + 1 \leq -19$:

$$
\boxed{
x \in \{4, 5, 6, 7, 8, 9, 10\}
}
$$

Broader Implications and Educational Significance

This problem exemplifies a classic method for solving linear inequalities, especially emphasizing the importance of:

- Proper algebraic manipulation, including awareness of the sign reversal when dividing by negative numbers.
- Combining multiple constraints (inequality and interval) to refine the solution set.
- Verifying solutions through substitution, a crucial step to confirm correctness.

It also highlights how simple inequalities can generate multiple solutions within a specified domain, illustrating the importance of systematic reasoning in mathematical problem-solving.

Additional Considerations and Extensions

1. Graphical Interpretation

Plotting the function $f(x) = -5x + 1$ and the line $f(x) = -19$ can visually demonstrate the solution set. The graph of $f(x)$ is a straight line with a negative slope, crossing the y-axis at 1, decreasing as x increases. The inequality $f(x) \leq -19$ corresponds to the region below or on the line $f(x) = -19$.

2. Generalization

For inequalities of the form $-ax + b \leq c$, where $a > 0$, the solution set can be found by:

$$
x \geq \frac{b - c}{a}
$$

and considering the domain constraints.

3. Real-World Applications

Understanding such inequalities has practical applications in fields such as economics (cost and revenue analysis), engineering (stress and load limits), and data science (threshold-based classifications).

Conclusion

The systematic exploration of the inequality $-5x + 1 \leq -19$ within the interval $3 \leq x \leq 10$ demonstrates a clear process of algebraic manipulation, logical reasoning, and validation. The integer solutions $\{4, 5, 6, 7, 8, 9, 10\}$ are derived through meticulous steps and verified against the original inequality, exemplifying a thorough approach beneficial for both educational and analytical contexts.

This detailed investigation underscores the importance of foundational algebraic techniques in solving inequalities and highlights the value of methodical reasoning in mathematical problem-solving.

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