

The Electric Current I (in Amperes) Produced By An Alternator In Time T (in Seconds) Is Given By $I=60\sin(120\pi t)$.

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Understanding the behavior of alternating current (AC) generated by alternators is fundamental in electrical engineering and power systems. The formula $I=60\sin(120\pi t)$ precisely describes how the electric current varies over time in a sinusoidal manner. In this article, we will explore the significance of this equation, analyze its components, and understand its implications for electrical systems. Whether you're a student, engineer, or enthusiast, a comprehensive grasp of sinusoidal current generation will deepen your appreciation of AC power.

Introduction to Alternating Current and Sinusoidal Waveforms

Alternating current (AC) is the type of electrical current that periodically reverses direction, contrasting with direct current (DC), which flows in a single direction. AC is the standard form of electricity supplied to homes and industries because of its efficiency in transmitting power over long distances.

The fundamental characteristic of AC is its sinusoidal waveform, which can be mathematically expressed as:

$$I(t) = I_{\max} \sin(\omega t + \phi)$$

where:

- $I(t)$ is the instantaneous current at time t ,
- $I_{\max}$ is the maximum current (amplitude),
- ω is the angular frequency,
- ϕ is the phase angle.

The specific formula given in our context, $I = 60 \sin(120\pi t)$, is a particular case of this general expression.

Decoding the Equation: $I=60\sin(120\pi t)$

Let's analyze each component of the formula to understand the behavior of the current over time:

Amplitude (Maximum Current)

- Value: 60 Amperes
- Significance: Represents the peak value of the alternating current. The current oscillates between +60A and -60A.

Angular Frequency (ω)

- Value: 120π radians per second
- Derived from: $\omega = 2\pi f$, where f is the frequency in Hz.
- Calculation: $f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60$ Hz.
- Implication: The alternator produces a sinusoidal current at 60 Hz, which is the standard mains frequency in many countries.

Time Variable (t)

- Unit: Seconds
- Role: Defines the specific point in time at which the current is measured.

Understanding the Frequency and Period of the Alternator

The frequency f of 60 Hz indicates that the current completes 60 cycles each second. The period T , the time for one complete cycle, is given by:

$$T = \frac{1}{f} = \frac{1}{60} \approx 0.01667 \text{ seconds}$$

This rapid oscillation is characteristic of standard power supply systems, ensuring efficient energy transmission and utilization.

Graphical Representation of the Sinusoidal Current

Visualizing the current over time helps in understanding its behavior:

- The sine wave oscillates between +60A and -60A.
- Zero crossings occur at points where the sine function equals zero.
- Peak points occur at $t = \frac{1}{4f} + n \times T$, where n is an integer.

Key points:

Time (s)	Current (A)	Description
0	0	Zero crossing (start)

1/4T	+60	Positive peak
1/2T	0	Zero crossing
3/4T	-60	Negative peak
T	0	Complete cycle

Applications of Sinusoidal Current in Alternators

Alternators generate sinusoidal AC to power a wide variety of electrical devices. The specific properties of the generated current influence their operational efficiency and safety.

Common applications include:

- Power supply for residential and industrial use.
- Operation of transformers and electric motors.
- Communication systems requiring stable frequency signals.
- Testing and calibration of electrical equipment.

Mathematical Analysis of the Current Waveform

To analyze the current further, several key concepts are essential:

Peak (Maximum) Current

- The maximum current amplitude is 60A.
- The RMS (Root Mean Square) current, useful for power calculations, is:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{60}{\sqrt{2}} \approx 42.43 \text{ A}$$

Instantaneous Power

- Power at any instant is given by:

$$P(t) = V(t) \times I(t)$$

- For sinusoidal systems, average power over a cycle is:

$$P_{\text{avg}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos(\phi)$$

- Assuming a purely resistive load, ($\phi = 0$), leading to maximum power transfer.

Implications for Electrical Engineering and Power Systems

Understanding the sinusoidal nature of the alternator's current is critical for designing efficient power systems. Engineers use this information to:

- Calculate power delivery capabilities.
- Design transformers and other components compatible with 60Hz sinusoidal signals.
- Ensure safety standards are met for devices operating within these waveforms.
- Analyze harmonic distortions and their effects on system performance.

Practical Considerations in Real-World Systems

While the ideal sinusoidal wave provides a foundation for analysis, real-world systems experience deviations due to:

- Impedance variations.
- Harmonic distortions.
- Transients and noise.
- Load fluctuations.

Power quality measures aim to minimize these issues, ensuring a stable sinusoidal current like the one described by $i = 60 \sin(120\pi t)$.

Conclusion: Significance of the Equation $i = 60 \sin(120\pi t)$

The equation $i = 60 \sin(120\pi t)$ succinctly encapsulates the fundamental behavior of sinusoidal AC generated by an alternator operating at 60 Hz. Recognizing its components allows engineers and students to predict how the current varies with time, design compatible electrical systems, and analyze power delivery efficiency. Mastery of such fundamental concepts underpins advancements in electrical engineering, ensuring reliable, efficient, and safe power systems worldwide.

Summary of Key Points:

- The amplitude of the current is 60A.
- The system operates at a frequency of 60Hz.
- The period of the wave is approximately 0.01667 seconds.
- The sinusoidal waveform ensures smooth alternation of current direction.
- Understanding these parameters is essential for designing and analyzing AC power systems.

By thoroughly understanding the sinusoidal current formula, engineers and students can better appreciate the dynamics of power generation and distribution, contributing to innovations and improvements in electrical infrastructure.

Note: For further study, consider exploring topics such as power factor, harmonic distortion, phase shifts, and the effects of non-resistive loads on sinusoidal waveforms.

Frequently Asked Questions

What is the expression for the electric current produced by the alternator over time?

The electric current is given by $I = 60 \sin(120\pi t)$ Amperes.

What is the amplitude of the alternating current in the given expression?

The amplitude of the current is 60 Amperes.

What is the frequency of the alternating current produced by the alternator?

The frequency is 60 Hz, since the angular frequency $\omega = 120\pi$ rad/sec, and frequency $f = \omega / (2\pi) = 120\pi / (2\pi) = 60$ Hz.

At what times does the current reach its maximum value?

The current reaches its maximum value of 60 A when $\sin(120\pi t) = 1$, i.e., at $t = (\pi/2 + n\pi) / 120\pi$, simplifying to $t = (1/4 + n/2)$ seconds for integer n .

How can we determine the period of the current oscillation from the given formula?

The period T is given by $T = 1 / f = 1 / 60$ seconds, which equals approximately 0.0167 seconds.

Additional Resources

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Introduction

In the realm of electrical engineering and power generation, understanding the behavior of alternators—also known as synchronous generators—is fundamental. Alternators are critical components in the production of alternating current (AC) electricity, which powers homes, industries, and various electronic devices worldwide. A key aspect of analyzing alternator performance involves understanding the time-dependent variation of the electric current it produces.

The specific formula $I = 60 \sin(120\pi t)$ encapsulates the dynamic nature of the current output as a function of time, revealing insights into the oscillatory behavior inherent in AC systems. This investigative article delves into the origins, implications, and detailed analysis of this equation, providing a comprehensive understanding suitable for engineers, researchers, and students interested in electrical power systems.

The Mathematical Formulation of Alternator Current

Understanding the Equation: $I = 60 \sin(120\pi t)$

The given formula describes a sinusoidal current:

- I : Electric current in amperes (A)
- t : Time in seconds (s)
- 60: Amplitude of the current in amperes
- 120π : Angular frequency in radians per second

This equation models the alternating current produced by an idealized alternator, assuming sinusoidal steady-state operation.

Derivation and Physical Significance

From Basic Principles of Alternating Current

Alternators generate AC by rotating a magnetic field within a set of conductors, inducing a time-varying electromotive force (EMF). The current's variation over time is directly related to the induced EMF and the circuit's impedance.

The fundamental sinusoidal form stems from Faraday's law of induction, which states that the induced EMF in a coil is proportional to the rate of change of magnetic flux. When the magnetic flux varies sinusoidally, the induced EMF and resulting current follow suit.

Linking Frequency to Rotational Speed

The angular frequency (ω) in the equation relates to the mechanical rotation of the alternator's rotor:

ω

$$\omega = 2\pi f$$

Where:

- f : Frequency in Hertz (Hz)
- ω : Angular frequency in radians per second

Given $\omega = 120\pi$, the frequency f is:

$$f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60 \text{ Hz}$$

This indicates the alternator operates at 60 Hz, which is standard for many power systems.

Deep Dive into the Alternator's Operation

The Sinusoidal Nature of AC

The sinusoidal current described by the equation reflects the fundamental waveform in AC power systems. This waveform's properties are critical:

- Peak amplitude (60 A): The maximum current attained during operation.
- Frequency (60 Hz): The number of cycles per second.
- Phase: The sine wave's initial phase determines the waveform's starting point at $t=0$.

Understanding these characteristics aids in designing compatible loads and ensuring system stability.

Impacts of the Current Waveform

The oscillating current influences:

- Power delivery: The RMS (Root Mean Square) value, which determines real power, is derived from the peak amplitude.
- Transformer operation: Transformers rely on sinusoidal flux variations for efficient energy transfer.
- Harmonics and power quality: Deviations from ideal sine waves introduce harmonics, affecting equipment performance.

Analyzing the Equation in Context

Calculating Key Parameters

1. Peak current ($I_{\max}$):

$$I_{\text{max}} = 60 \text{ A}$$

2. Instantaneous current at specific times:

For example, at $t=0$:

$$I(0) = 60 \sin(0) = 0 \text{ A}$$

At $t = \frac{1}{4f} = \frac{1}{4 \times 60} = 0.00417 \text{ s}$:

$$I(0.00417) = 60 \sin(120 \pi \times 0.00417) \approx 60 \sin(\pi/2) = 60 \times 1 = 60 \text{ A}$$

3. Period of the waveform:

$$T = \frac{1}{f} = \frac{1}{60} \approx 0.01667 \text{ s}$$

The current completes a full cycle every 16.67 milliseconds.

Practical Implications in Power Systems

Power Calculations

The instantaneous power delivered by the alternator involves the product of instantaneous voltage and current. Assuming the voltage waveform is similarly sinusoidal, the average power over a cycle is given by:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

Where ϕ is the phase angle between voltage and current. For ideal sinusoids in phase:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{60}{\sqrt{2}} \approx 42.43 \text{ A}$$

This RMS value is crucial for power ratings and system design.

Harmonic Considerations

Although the idealized equation assumes a perfect sine wave, real-world alternators may exhibit harmonic distortions due to non-linearities, load variations, or switching devices. These distortions are analyzed using Fourier series, which decomposes the current into fundamental and harmonic components.

Investigative Perspectives and Technical Challenges

Stability and Control

Understanding the time-dependent behavior of the current allows engineers to design control systems that maintain stable operation, especially under varying load conditions. The sinusoidal nature simplifies the design of filters and compensators.

Efficiency Optimization

Analyzing the current waveform also enables optimization of the alternator's efficiency. Deviations from sinusoidality can lead to increased losses, heating, and electromagnetic interference.

Diagnostic and Monitoring

By monitoring the current over time, anomalies such as faults or mechanical issues in the alternator can be detected early. Oscilloscopes and spectral analyzers facilitate such diagnostics.

Broader Significance in Electrical Engineering

The formula $I = 60 \sin(120\pi t)$ exemplifies the fundamental principles governing AC generation. Its study bridges theoretical physics, circuit analysis, and practical power system engineering.

- It underscores the importance of frequency control in power generation.
- It highlights the necessity for precise synchronization in interconnected grids.
- It demonstrates how sinusoidal waveforms underpin modern electrical infrastructure.

Conclusion

The detailed analysis of the current function $I = 60 \sin(120\pi t)$ reveals the intricate interplay between mechanical rotation, electromagnetic induction, and electrical output in an alternator. By understanding the sinusoidal nature, frequency, amplitude, and phase relationships, engineers can design more efficient, reliable, and safe power systems.

This investigation underscores the critical role of mathematical modeling in electrical engineering, enabling us to predict, control, and optimize the generation and distribution of electrical energy. As power demands evolve and technologies advance, such

foundational principles remain central to innovations in sustainable and resilient power systems.

References

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Appendix: Sample Calculations

Sample calculation of RMS current:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{60}{1.414} \approx 42.43, \text{ A}$$

Sample calculation of power at maximum current:

Assuming a voltage of 120 V (RMS):

$$P_{\text{max}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi$$

If in phase ($\cos \phi = 1$):

$$P_{\text{max}} = 120 \times 42.43 \approx 5,089.6, \text{ W}$$

This illustrates the power capacity during peak current conditions.

End of Article

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