The Explicit Rule For A Sequence And One Of The Specific Terms Is Given. Find The Position Of The Giventerm.f(n)

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Understanding sequences and their explicit formulas is a fundamental concept in mathematics, especially in algebra and calculus. When dealing with a sequence, one of the most common problems involves identifying the position of a specific term within that sequence, given an explicit rule and one of its terms. This process is essential for solving real-world problems, analyzing mathematical patterns, and advancing in higher-level mathematics.

In this comprehensive guide, we will explore the steps to find the position of a given term in a sequence when the explicit formula is known, delve into various types of sequences, and provide practical examples to enhance understanding.

Understanding Sequences and Explicit Formulas

Before tackling the problem, it's vital to understand what sequences are and how explicit formulas are used to describe them.

What Is a Sequence?

A sequence is an ordered list of numbers following a specific pattern or rule. Each number in the sequence is called a term, and the position of each term is its index or term number, typically denoted as n.

For example:
- 2, 4, 6, 8, 10, ... (sequence of even numbers)
- 1, 3, 5, 7, 9, ... (sequence of odd numbers)
- 3, 6, 12, 24, 48, ... (geometric sequence)

Explicit Formula vs. Recursive Formula

- Recursive Formula expresses each term based on previous terms. For example, $a_{n} = a_{n-1} + d$.
- Explicit Formula directly calculates the nth term using n without referring to previous terms. For example, $a_{n} = 2n$.

The explicit formula is particularly useful for directly finding the position of a term because it simplifies the process to solving an algebraic equation.

Types of Sequences and Their Explicit Rules

Sequences can be classified broadly into arithmetic, geometric, and other special types, each with its own explicit rule.

Arithmetic Sequences

- Defined by a common difference d.
- Explicit formula: $f(n) = a_1 + (n - 1)d$
- where a_1 is the first term.

Geometric Sequences

- Defined by a common ratio r.
- Explicit formula: $f(n) = a_1 \times r^{n-1}$

Other Sequences

- Could follow polynomial, exponential, or factorial patterns.
- Their explicit formulas vary and may involve more complex expressions.

Finding the Position of a Term Given Its Value

Given:
- The explicit rule $f(n)$
- A specific term value $f(n) = T$

Goal:
- Find the term's position n

Approach:
1. Write down the explicit formula.
2. Set the formula equal to the given term value.
3. Solve the resulting algebraic equation for n.

This process may involve algebraic manipulation, solving quadratic equations, logarithms, or other methods depending on the sequence type.

Step-by-Step Methodology to Find the Position of a Given Term

Let's explore a systematic approach:

Step 1: Identify the Sequence Type and Write Its Explicit Formula

Determine whether the sequence is arithmetic, geometric, or of another type, then write its explicit formula.

Step 2: Substitute the Given Term Value into the Formula

Replace $f(n)$ with the known term value T.

Step 3: Solve for n

Rearrange the equation to solve for n. This may involve:
- Basic algebra
- Logarithmic operations (for exponential or geometric sequences)
- Quadratic formula (if quadratic in form)

Step 4: Verify the Solution

Ensure that the calculated n is a positive integer, as sequence positions are whole numbers.

Step 5: Interpret the Result

- If n is an integer, it indicates the position of the term.
- If n is not an integer, check for possible approximations or whether the term exists in the sequence.

Practical Examples and Solutions

Let's consider some concrete examples to illustrate the process.

Example 1: Arithmetic Sequence

Given:
- Sequence: 5, 8, 11, 14, ...
- Explicit formula: $f(n) = 5 + (n - 1) \times 3$
- Find the position of the term $T = 23$.

Solution:
1. Write the explicit formula: $f(n) = 5 + 3(n - 1)$
2. Set $f(n) = 23$:

$$
23 = 5 + 3(n - 1)
$$

3. Solve for n:

$$
23 - 5 = 3(n - 1)
$$

$$
18 = 3(n - 1)
$$

$$
n - 1 = \frac{18}{3} = 6
$$

$$
n = 7
$$

Result:
- The term 23 is at position 7 in the sequence.

Example 2: Geometric Sequence

Given:
- Sequence: 3, 6, 12, 24, ...
- Explicit formula: $f(n) = 3 \times 2^{n-1}$
- Find the position of the term $T = 96$.

Solution:
1. Write the explicit formula: $f(n) = 3 \times 2^{n-1}$
2. Set $f(n) = 96$:

$$
96 = 3 \times 2^{n-1}
$$

3. Solve for n:

$$
2^{n-1} = \frac{96}{3} = 32
$$

$$
2^{n-1} = 2^5
$$

$$
n - 1 = 5
$$

$$
n = 6
$$

Result:

- The term 96 is at position 6.

Handling More Complex Sequences

While arithmetic and geometric sequences are straightforward, some sequences involve quadratic, polynomial, or exponential growth, requiring more advanced techniques.

Quadratic Sequences

- Explicit formulas involve quadratic expressions: $f(n) = an^2 + bn + c$.
- To find n, set $f(n) = T$ and solve quadratic equations:

$$
an^2 + bn + c = T
$$

- Use quadratic formula:

$$
n = \frac{-b \pm \sqrt{b^2 - 4a(c - T)}}{2a}
$$

- Choose the positive integer solution as the sequence position.

Sequences Defined by Logarithms or Exponentials

- Use logarithmic properties to solve for n:

$$
T = a \times r^{n-1}
$$

$$
\Rightarrow n = 1 + \frac{\log(T/a)}{\log r}
$$

- Verify whether n is an integer.

Common Challenges and Tips

- Approximate Solutions: Sometimes, solving for n yields a non-integer. Consider whether the sequence contains the term or if an approximation suffices.
- Sequence Domain: Ensure the n value makes sense within the sequence's domain (positive integers).
- Multiple Solutions: Quadratic or higher-degree equations may have multiple solutions; select the positive integer that fits sequence criteria.

- Sequence Context: Always verify the found position by substituting back into the explicit formula.

Summary and Key Takeaways

- When given an explicit rule and a specific term, solving for the position involves algebraic manipulation.
- Recognize the sequence type to apply appropriate formulas.
- For arithmetic sequences, linear equations suffice.
- For geometric sequences, logarithms are often necessary.
- Complex sequences may require solving quadratic or higher-degree equations.
- Always verify solutions for consistency with the sequence.

Final Thoughts

Mastering the method to find the position of a term in a sequence given its explicit rule and value is a fundamental skill in mathematics. It enhances analytical thinking, problem-solving abilities, and provides insights into the underlying patterns of numerical sequences. Practice with different sequence types and apply the outlined steps to develop confidence and proficiency in tackling such problems.

By understanding the explicit formulas and solving the corresponding equations, you can efficiently determine the position of any term within a sequence, an essential step in advanced mathematics, data analysis, and various scientific applications.

Frequently Asked Questions

How do I find the position of a specific term in a sequence with an explicit rule when that term is given?

To find the position, substitute the known term value into the explicit formula and solve for n, which represents the position in the sequence.

What is the general approach to determine the term number in a sequence when the explicit formula and a specific term are known?

Set the explicit formula equal to the given term value and solve for n to find the position of that term in the sequence.

Can you give an example of finding the position of a term in an arithmetic sequence if the explicit formula and term value are known?

Yes. For example, if the explicit formula is $f(n) = 3n + 2$ and the term value is 14, set $3n + 2 = 14$, solve for n: $n = (14 - 2)/3 = 4$. So, the term 14 is at position 4.

What should I do if solving for n results in a non-integer value in finding the position of a term?

If n is not an integer, it indicates that the given term does not appear at a whole-number position in the sequence, meaning the term isn't part of the sequence at an integer index.

Is it necessary for the sequence's explicit rule to be linear when finding the position of a given term?

No, the method applies to any explicit rule, linear or nonlinear, by setting the formula equal to the term value and solving for n, provided the formula is invertible.

How do I handle sequences with explicit formulas involving exponents when finding the position of a specific term?

Set the explicit formula equal to the given term and solve for n using algebraic methods appropriate for the function, such as logarithms for exponential functions.

What are common mistakes to avoid when finding the position of a term given its value and an explicit rule?

Common mistakes include incorrect algebraic manipulation, forgetting to verify if the resulting n is a positive integer, and not checking whether the solution makes sense within the sequence context.

Can the method of finding the position be used for sequences defined recursively instead of explicitly?

No, for recursively defined sequences, you typically need to find a closed-form explicit formula first. Once you have the explicit rule, you can then find the position of a specific term.

Additional Resources

The Explicit Rule For A Sequence And One Of The Specific Terms Is Given. Find The Position Of The Given Term f(n)

Introduction

Sequences are fundamental constructs in mathematics, with applications spanning across algebra, calculus, computer science, and beyond. They serve as a bridge between discrete and continuous mathematics, providing a framework to analyze patterns, convergence, and divergence. A common challenge encountered when working with sequences involves determining the position (or index) of a specific term within the sequence, especially when the sequence’s explicit rule is known but the term’s index is not. Conversely, sometimes the sequence's explicit formula is unknown, but a particular term is given, and the goal is to find its position.

This investigative article explores the problem: Given an explicit rule for a sequence and one of its specific terms, how can we find the position n of that term $f(n)$? We will examine foundational methods, theoretical underpinnings, practical approaches, and illustrative examples. The discussion aims to serve as a comprehensive guide for students, educators, and researchers dealing with similar sequence problems.

Understanding the Core Problem

Defining the Sequence and Known Data

Suppose we have a sequence $\{f(n)\}$, where:

- The explicit rule (or formula) for the n-th term is known: $f(n) = \text{expression involving } n$.
- A specific term T is given: $f(n_0) = T$.
- The goal: Find the index n_0 such that the sequence’s n_0-th term equals T.

Typical Scenarios

1. Explicit Formula Known, Term Given: Find n such that $f(n) = T$.

2. Sequence Defined Recursively, Term Given: Find n satisfying the recurrence condition, often requiring solving recurrence relations.

3. Sequence with Multiple Parameters: When the formula involves constants, additional information may be necessary to determine the index.

Significance in Mathematical Analysis

Locating a term's position within a sequence aids in understanding the sequence's growth, behavior, and the distribution of its terms. It is particularly relevant in:

- Number theory (e.g., identifying the position of special numbers)
- Algorithm analysis (e.g., determining the iteration count for an algorithm)
- Pattern recognition and sequence classification

Theoretical Foundations and Methods

1. Direct Algebraic Inversion

The most straightforward approach involves algebraically solving the explicit formula for n given the value T:

$$
f(n) = T \quad \Rightarrow \quad n = f^{-1}(T)
$$

Example: Consider an arithmetic sequence:

$$
f(n) = a + (n - 1)d
$$

Given T, solve:

$$
a + (n - 1)d = T \quad \Rightarrow \quad n = \frac{T - a}{d} + 1
$$

If n is an integer, and the right side evaluates to an integer, then n is the position of the term T.

---

2. Inverting Polynomial or Rational Formulas

When the explicit rule involves polynomial expressions, inversion may require solving polynomial equations:

- Quadratic: $an^2 + bn + c = T$, solve for n using the quadratic formula.
- Higher-degree polynomials: employ algebraic techniques or numerical methods.

Example:

Suppose

$$
f(n) = n^2 + 3n + 2
$$

Given T, find n:

$$
n^2 + 3n + 2 = T
$$

$$
n^2 + 3n + (2 - T) = 0
$$

Solve:

$$
n = \frac{-3 \pm \sqrt{9 - 4(1)(2 - T)}}{2}
$$

Choose the solution(s) that yield positive integers.

---

3. Logarithmic and Exponential Sequences

Sequences involving exponential or logarithmic functions often require taking logarithms:

Example:

$$
f(n) = a \cdot r^{n}
$$

Given T, solve:

$$
a \cdot r^{n} = T \quad \Rightarrow \quad r^{n} = \frac{T}{a}
$$

$$
n = \log_{r}\left(\frac{T}{a}\right)
$$

Determine whether n is an integer, and if not, whether the sequence terms are defined for non-integer indices.

Practical Approaches and Considerations

1. Checking for Validity of the Solution

- After solving for n, verify whether n is an integer.
- Confirm the domain restrictions; sequences often are defined only for positive integers.
- Consider whether the sequence is strictly increasing/decreasing to understand the uniqueness of the solution.

2. Handling Non-Integer Solutions

In some cases, solving the explicit formula yields non-integer n. Since sequence indices are often integers, the term T may not correspond to any term in the sequence. Alternatively, the sequence might be extended to real indices, but in typical discrete sequences, only integer n are valid.

3. Multiple Solutions

Quadratic or higher-degree equations may produce multiple solutions. Determine which solutions are valid based on the sequence’s domain.

4. Numerical Methods

When explicit inversion is complex or impossible analytically, numerical methods such as:

- Newton-Raphson method
- Bisection method
- Secant method

can approximate the value of n.

Illustrative Examples

Example 1: Arithmetic Sequence

Given:

$$
f(n) = 7 + 3(n - 1)
$$

Find the position n of the term $T = 22$.

Solution:

$$
7 + 3(n - 1) = 22
$$

$$
3(n - 1) = 15
$$

$$
n - 1 = 5
$$

$$
n = 6
$$

Thus, the term 22 is at position 6.

Example 2: Quadratic Sequence

Given:

$$
f(n) = n^2 + 2n
$$

Find n for $T = 20$.

Solution:

$$
n^2 + 2n = 20
$$

$$
n^2 + 2n - 20 = 0
$$

Apply quadratic formula:

$$
n = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times (-20)}}{2}
$$

$$
n = \frac{-2 \pm \sqrt{4 + 80}}{2} = \frac{-2 \pm \sqrt{84}}{2}
$$

$$
n = \frac{-2 \pm 2\sqrt{21}}{2} = -1 \pm \sqrt{21}
$$

Since $\sqrt{21} \approx 4.58$:

- $n \approx -1 + 4.58 = 3.58$ (not an integer)
- $n \approx -1 - 4.58 = -5.58$ (discarded, since sequence indices are positive)

No integer solution exists; hence, $T=20$ does not occur as a sequence term at an integer position.

Example 3: Geometric Sequence

Given:

$$
f(n) = 3 \times 2^{n-1}
$$

Find n for $T=24$.

Solution:

$$
3 \times 2^{n-1} = 24
$$

$$
2^{n-1} = 8
$$

$$
2^{n-1} = 2^3
$$

$$
n - 1 = 3
$$

$$
n = 4
$$

The term 24 appears at position 4.

Advanced Topics and Edge Cases

1. Sequences Defined by Recurrence Relations

When the explicit formula is not directly available, but the sequence is defined recursively, solving for n involves:

- Deriving the explicit formula (if possible)
- Solving the recurrence relation for the explicit form
- Then applying the algebraic inversion methods described above

2. Sequences with Piecewise Definitions

For sequences defined differently over various intervals, identifying the position of a term involves:

- Determining which segment the term belongs to
- Solving the corresponding explicit formula within that segment

3. Infinite and Non-Standard Sequences

In sequences involving limits or asymptotic behavior, the concept of position may be extended or reinterpreted, especially in sequences approaching a finite limit.

Summary and Best Practices

- Identify the explicit formula for $f(n)$. The inversion process hinges on this step.
- Solve algebraically for n in terms of T.
- Verify the solution: check whether n is a positive integer.
- Use numerical methods if the explicit formula is complex or not invertible analytically.
- Assess domain restrictions and the validity of solutions within the sequence’s context.
- Consider

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