The Fundamental Period Of The Signalx(t)=exp(j(7/3)n)is B. Find Whether The Signal Is Periodic Or Notx(t)=2cos(10t+1)sin(4t1)c.

The Fundamental Period Of The Signalx(t)=exp(j(7/3)n)is B. Find Whether The Signal Is Periodic Or Notx(t)=2cos(10t+1)sin(4t1)c.

Understanding the fundamental period of signals is essential in signal processing, communications, and electronics. In this comprehensive guide, we analyze two distinct signals: the discrete-time complex exponential signal $x(n) = e^{j \frac{7}{3} n}$ and the continuous-time real-valued signal $x(t) = 2 \cos(10 t + 1) \sin(4 t)$. Our goal is to determine the fundamental period of the first signal and assess whether the second signal is periodic, providing detailed explanations, mathematical derivations, and practical insights.

Analyzing the Fundamental Period of the Discrete-Time Signal $x(n) = e^{j \frac{7}{3} n}$

Understanding Discrete-Time Complex Exponentials

The discrete-time complex exponential signals are of fundamental importance in signal processing because they form the basis for Fourier analysis and spectral representation. The general form:

$$
x(n) = e^{j \omega n}
$$

where:

- ω is the angular frequency (in radians per sample),
- n is the discrete-time index.

Such signals are inherently periodic if ω is a rational multiple of 2π. The key to finding the fundamental period is understanding when the exponential repeats itself.

Determining the Fundamental Period N_0

For the signal:

$$
x(n) = e^{j \frac{7}{3} n}
$$

the angular frequency:

$$
\omega = \frac{7}{3}
$$

is in radians per sample.

The periodicity condition for a discrete-time complex exponential is:

$$
x(n + N_0) = x(n) \quad \text{for all } n
$$

which implies:

$$
e^{j \omega (n + N_0)} = e^{j \omega n}
$$

or:

$$
e^{j \omega N_0} = 1
$$

Since $e^{j \theta} = 1$ when $\theta = 2 \pi k$, where k is an integer, we have:

$$
\omega N_0 = 2 \pi k
$$

Plugging in $\omega = \frac{7}{3}$:

$$
\frac{7}{3} N_0 = 2 \pi k
$$

\]

Rearranged as:

\[
N_0 = \frac{2 \pi k \times 3}{7}
\]

To find the fundamental period \(N_0 \), we seek the smallest positive integer \(N_0 \) such that this equation holds for some integer \(k \).

Finding the Minimal \(N_0 \)

Rearranged:

\[
N_0 = \frac{6 \pi k}{7}
\]

Since we need \(N_0 \) to be an integer, and \(\pi \) is irrational, the above expression suggests that the exponential will be periodic if and only if \(\omega \) is a rational multiple of \(2 \pi \).

Alternatively, consider the frequency ratio:

\[
\frac{\omega}{2 \pi} = \frac{7/3}{2 \pi} = \frac{7}{3 \times 2 \pi}
\]

which is irrational because \(\pi \) is irrational. Therefore, the exponential \(e^{j \frac{7}{3} n} \) is not periodic in the discrete-time domain because there is no finite \(N_0 \) satisfying the periodicity condition.

Conclusion:
The discrete-time complex exponential \(x(n) = e^{j \frac{7}{3} n} \) is not periodic because its frequency is an irrational multiple of \(2\pi \).

Analyzing the Periodicity of $x(t) = 2 \cos(10 t + 1) \sin(4 t)$

Understanding Continuous-Time Signals and Periodicity

A continuous-time signal $x(t)$ is periodic if there exists a positive real number T such that:

$$
x(t + T) = x(t) \quad \text{for all } t
$$

The fundamental period T_0 is the smallest such positive T.

In the case of products of sinusoidal functions, the periodicity depends on the individual frequencies involved.

Expressing the Signal Using Trigonometric Identities

The given signal:

$$
x(t) = 2 \cos(10 t + 1) \sin(4 t)
$$

can be simplified using the product-to-sum identities:

$$
2 \cos A \sin B = \sin(A + B) - \sin(A - B)
$$

Applying this, we obtain:

$$
x(t) = \sin(10 t + 1 + 4 t) - \sin(10 t + 1 - 4 t)
$$

which simplifies to:

$$
x(t) = \sin(14 t + 1) - \sin(6 t + 1)
$$

Now, the signal is expressed as the difference of two sinusoidal functions:

$$
x(t) = \sin(14 t + 1) - \sin(6 t + 1)
$$

Determining Periods of Individual Components

The two sinusoidal components are:

1. $\sin(14 t + 1)$
2. $\sin(6 t + 1)$

The periods of these functions are:

$$
T_1 = \frac{2 \pi}{14} = \frac{\pi}{7}
$$
$$
T_2 = \frac{2 \pi}{6} = \frac{\pi}{3}
$$

Since phase shifts (the additional "+ 1") do not affect periodicity, these are the fundamental periods of the individual sinusoids.

Finding the Fundamental Period of $x(t)$

The overall periodicity of $x(t)$ depends on whether these two sinusoidal components are commensurate—that is, whether their periods are rational multiples of each other.

To check:

$$
\frac{T_1}{T_2} = \frac{\pi / 7}{\pi / 3} = \frac{3}{7}
$$

which is rational. Therefore, the two components are commensurate, and the combined signal $x(t)$ is periodic with a fundamental period equal to the least common multiple (LCM) of T_1 and T_2.

The least common multiple of T_1 and T_2:

$$
T_0 = \text{LCM}(T_1, T_2)
$$

Since:

$$
T_1 = \frac{\pi}{7}
$$
$$
T_2 = \frac{\pi}{3}
$$

the fundamental period T_0 is:

$$
T_0 = \text{LCM} \left(\frac{\pi}{7}, \frac{\pi}{3} \right) = \pi \times \text{LCM} \left(\frac{1}{7}, \frac{1}{3} \right)
$$

Expressed as:

$$
T_0 = \pi \times \frac{\text{LCM}(1/7, 1/3)}{1}
$$

But more straightforwardly, the period of the sum of two sinusoids with periods T_1 and T_2 is:

$$
T_0 = \text{LCM}(T_1, T_2) = \text{smallest positive } T \text{ such that } T = m T_1 = n T_2, \text{ for integers } m, n
$$

Calculating:

$$
m T_1 = n T_2
$$
$$
m \times \frac{\pi}{7} = n \times \frac{\pi}{3}
$$

Dividing both sides by π:

$$
\frac{m}{7} = \frac{n}{3}
$$

Cross-multiplied:

$$
3 m = 7 n
$$

Since m, n are integers, the minimal solution occurs at:

$$
m = 7, \quad n = 3
$$

This gives:

$$
T_0 = m T_1 = 7 \times \frac{\pi}{7} = \pi
$$

or equivalently:

$$
T_0 = n T_2 = 3 \times \frac{\pi}{3} = \pi
$$

Conclusion:
The fundamental period of $x(t)$ is:

$$
\boxed{
T_0 = \pi
}
$$

The signal $x(t)$ is periodic with a fundamental period of \(\

Frequently Asked Questions

What is the fundamental period of the discrete-time signal x(t) = exp(j(7/3)n)?

Since the signal involves a complex exponential with a rational multiple of n, the fundamental period T can be found by ensuring (7/3) T is an integer. Therefore, T = 3, making the signal periodic with a fundamental period of 3.

Is the discrete-time signal x(t) = exp(j(7/3)n) periodic?

Yes, because (7/3) is a rational number, the exponential signal is periodic with period T = 3.

How do you determine if the continuous-time signal x(t) = 2cos(10t + 1)sin(4t) is periodic?

To determine the periodicity, analyze the individual frequencies within the signal. Since it is a product of cosines and sines, express it as a sum of cosines using product-to-sum identities, then find the fundamental period based on the combined frequencies.

Is the signal x(t) = 2cos(10t + 1)sin(4t) periodic?

Yes, the signal is periodic because both component frequencies (10 rad/sec and 4 rad/sec) are rational multiples of pi, leading to a common fundamental period.

What is the fundamental period of x(t) = 2cos(10t + 1)sin(4t)?

Using trigonometric identities, the combined frequencies lead to a fundamental period T = 2π / gcd(10, 4) = 2π / 2 = π.

How can the periodicity of a product of sinusoidal signals be determined?

By expressing the product as a sum of cosines with different frequencies and finding the least common multiple (LCM) of their individual periods, the overall fundamental period can be determined.

Can the signal x(t) = 2cos(10t + 1)sin(4t) be considered a sampled version of a discrete-time signal?

No, this is a continuous-time signal involving real-valued sinusoids. Sampling considerations are separate and depend on the sampling frequency relative to the signal's bandwidth.

What is the significance of rational vs. irrational frequency ratios in determining periodicity?

If the ratio of frequencies is rational, the signal is periodic with a finite fundamental period. If irrational, the signal is aperiodic and does not have a finite fundamental period.

How does the phase shift in cos(10t + 1) affect the periodicity of the signal?

Phase shifts do not affect the period of sinusoidal signals. The periodicity depends on the frequency components, not the phase.

Additional Resources

Fundamental Period of Signals: A Comprehensive Analysis of x(t) = exp(j(7/3) n) and x(t) = 2cos(10t + 1) sin(4t)

Introduction

In the realm of signal processing and systems analysis, understanding the periodicity of signals is foundational. Whether dealing with discrete or continuous signals, identifying the period, if it exists, is crucial for applications such as Fourier analysis, modulation, sampling, and filtering. This detailed review focuses on two specific signals:

1. $x(n) = e^{j \frac{7}{3} n}$, a discrete-time complex exponential signal.
2. $x(t) = 2 \cos(10t + 1) \sin(4t)$, a continuous-time real-valued signal.

We will explore the concept of fundamental period, determine whether each signal is periodic, and understand the underlying principles that govern their behavior.

Understanding Periodicity in Signals

Definition of Periodicity

A signal $x(t)$ (continuous-time) or $x(n)$ (discrete-time) is said to be periodic if there exists a positive constant T (for continuous-time) or N (for discrete-time), such that:

- For continuous-time:

$$
x(t + T) = x(t) \quad \forall t
$$

- For discrete-time:

$$
x(n + N) = x(n) \quad \forall n
$$

The smallest positive T or N for which the above holds is called the fundamental period.

Key Concepts

- Complex exponential signals are inherently periodic when their frequency components are rational multiples of the fundamental frequency.
- Trigonometric functions such as sine and cosine are periodic, but their product may or may not be periodic depending on their frequencies.
- For discrete-time signals, periodicity depends on the ratio of frequencies being rational; irrational ratios imply a non-periodic signal.

Analysis of the Discrete-Time Signal: $x(n) = e^{j \frac{7}{3} n}$

Nature of the Signal

This is a complex exponential sequence, a fundamental building block in digital signal processing. The general form:

$$
x(n) = e^{j \omega n}
$$

where $\omega = \frac{7}{3}$ in this case.

Determining Periodicity

To find the fundamental period N_0, we need to find the smallest positive integer N satisfying:

\[

x(n + N) = x(n) \quad \Rightarrow \quad e^{j \omega (n + N)} = e^{j \omega n}
\]

which simplifies to:

\[
e^{j \omega N} = 1
\]

This condition implies:

\[
\omega N = 2 \pi k \quad \text{for some integer } k
\]

or

\[
\frac{7}{3} N = 2 \pi k
\]

But note that \(\omega \) is in radians per sample, so the exponential's periodicity depends on whether \(\omega N \) is an integer multiple of \(2\pi \).

Expressing \(\omega \) in terms of \(2\pi \)

Since the argument of the exponential is in radians, the periodicity condition becomes:

\[
e^{j \frac{7}{3} N} = 1
\]

which is true when:

\[
\frac{7}{3} N = 2 \pi k
\]

for some integer \(k \).

But because \(\frac{7}{3} \) is a rational number, we can express this as:

\[

\frac{7}{3} N = 2 \pi k
\]

or equivalently,

\[
N = \frac{2 \pi k \times 3}{7}
\]

However, since the exponential is in discrete samples, and the periodicity condition involves the argument being a multiple of \(2\pi \), the key is to find the smallest positive integer \(N \) such that:

\[
\frac{7}{3} N = 2 \pi k
\]

but because \(2 \pi \) and \(\frac{7}{3} \) are incompatible in terms of rationality, it is more straightforward to consider the discrete-time frequency.

Discrete-Time Angular Frequency

The normalized discrete-time frequency:

\[
\omega_0 = \frac{7}{3}
\]

is in radians per sample. The periodicity condition for discrete-time complex exponentials is:

\[
e^{j \omega_0 N} = 1
\]

which is equivalent to:

\[
\omega_0 N = 2 \pi k
\]

Given \(\omega_0 = \frac{7}{3} \), the periodicity condition becomes:

\[
\frac{7}{3} N = 2 \pi k

\]

Dividing both sides by \(2\pi \):

\[
\frac{7}{3} \times \frac{N}{2 \pi} = k
\]

But in digital signal processing, the fundamental period exists if and only if \(\omega_0 \) is a rational multiple of \(2\pi \), i.e.,

\[
\frac{\omega_0}{2\pi} = \frac{7/3}{2\pi} = \frac{7}{3 \times 2 \pi}
\]

which is irrational because \(\pi \) is irrational, unless \(\frac{7}{3} \) is rational (which it is), but the key is whether the ratio of frequencies leads to a repeating sequence.

Alternatively, the periodicity condition simplifies to:

\[
e^{j \frac{7}{3} N} = 1 \Rightarrow \frac{7}{3} N = 2 \pi k
\]

which is only possible if \(\frac{7}{3} N \) is an integer multiple of \(2 \pi \).

Given that \(\frac{7}{3} \) is rational, the discrete-time exponential is periodic if \(\frac{\omega_0}{2 \pi} \) is rational, which it is, and the period is related to the least common multiple of the numerator and denominator.

Calculating the Fundamental Period

Given:

\[
\omega_0 = \frac{7}{3}
\]

and the periodicity condition:

\[
e^{j \omega_0 N} = 1
\]

which implies:

$$
\omega_0 N = 2 \pi k
$$

Express ω_0 in terms of 2π:

$$
\frac{7}{3} = \frac{2 \pi M}{N}
$$

for some integers M, N. But more straightforwardly, the discrete-time exponential $e^{j \omega_0 n}$ is periodic if:

$$
\frac{\omega_0}{2 \pi} = \frac{p}{q}
$$

where p, q are integers with no common factors, i.e., a rational number. Here:

$$
\frac{\omega_0}{2 \pi} = \frac{7/3}{2 \pi}
$$

which is irrational because 2π is irrational.

Conclusion: Since $\frac{\omega_0}{2 \pi}$ is irrational, the sequence $x(n) = e^{j \frac{7}{3} n}$ is not periodic.

Summary for $x(n) = e^{j \frac{7}{3} n}$

- The discrete-time complex exponential is not periodic because its normalized frequency $\frac{\omega_0}{2\pi}$ is irrational.
- The absence of a finite N satisfying $e^{j \omega_0 N} = 1$ confirms this.
- Therefore, $x(n)$ is a non-periodic sequence in the discrete domain.

Analysis of the Continuous-Time Signal: $x(t) = 2 \cos(10t + 1) \sin(4t)$

Nature of the Signal

This is a product of two trigonometric functions:

\[
x(t) = 2 \cos(10t + 1) \sin(4t)
\]

which involves a cosine and a sine function with different angular frequencies. To analyze its periodicity, it helps to express this product in a more manageable form using trigonometric identities.

Simplification Using Product-to-Sum Identities

Recall the identities:

- \(\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)] \)

Applying this to \(x(t) \):

\[
x(t) = 2 \times \cos(10t + 1) \times \sin(4t) = 2 \times \frac{1}{2} [\sin

The Fundamental Period Of The Signalx T Exp J 7 3 N Is B Find Whether The Signal Is Periodic Or Notx T 2cos 10t 1 Sin 4t1 C

The Fundamental Period Of The Signalx T Exp J 7 3 N Is B Find Whether The Signal Is Periodic Or Notx T 2cos 10t 1 Sin 4t1 C

Related Articles

- Do Only 2 Question And Give Brief Answer Its 4 Marksquestion*What Is The Importance Of Implementing Automatedsystems
- Approximately What Percentage Of The Body's Store Of Vitamin K Is Derived From GI Tract Bacterial Synthesis?a.Less
- For The Following Exercises, Find The Local And Absolute Minima And Maxima For The Functions Over ([infinity],[infinity]).

Back to Home