The graph of a quadratic polynomial p(x) passes through the points (-6,0), (0, -30), (4,-20) and (6,0). The zeroes of the polynomial are A) - 6,0 B) 4, 6 C) - 30,-20 D) - 6,6

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Understanding the Problem

When analyzing a quadratic polynomial, especially in the context of its graph passing through specific points, it is essential to understand the relationship between the points and the roots (or zeroes) of the polynomial. The problem provides four points:

- (-6, 0)
- (0, -30)
- (4, -20)
- (6, 0)

The question asks: What are the zeroes of the polynomial? and provides four options.

Key Concepts in Quadratic Polynomials

Definition and General Form

A quadratic polynomial p(x) can be expressed as:

\[p(x) = ax^2 + bx + c \]

where:

- a, b, c are constants, with $a \neq 0$.

Roots (Zeroes) of the Polynomial

- Roots (or zeroes) are the values of x where $p(x) = 0$.
- The roots are the x-intercepts of the graph.
- The quadratic can be factored as:

$$p(x) = a(x - r_1)(x - r_2)$$

where r_1 and r_2 are the roots.

Vertex and Symmetry

- The axis of symmetry passes through the vertex.
- For roots r_1 and r_2, the axis of symmetry is at:

$$x = \frac{r_1 + r_2}{2}$$

Analyzing the Given Points

Let's examine the given points:

Point	Coordinates	Interpretation
(-6, 0)	$x = -6$, $p(x) = 0$	Zero at $x = -6$
(0, -30)	$x = 0$, $p(x) = -30$	Point on the parabola
(4, -20)	$x = 4$, $p(x) = -20$	Point on the parabola
(6, 0)	$x = 6$, $p(x) = 0$	Zero at $x = 6$

From the points, it's clear that:

- The parabola passes through points where $x = -6$ and $x = 6$ with $p(x) = 0$. These are likely the roots.
- The points at $x=0$ and $x=4$ are not roots since $p(x) \neq 0$ there.

Identifying the Roots of the Polynomial

Zeroes Based on the Points

Since $p(-6) = 0$ and $p(6) = 0$, it strongly suggests that the roots are at $x = -6$ and $x = 6$. This aligns with option A and D:

- A) -6, 0
- B) 4, 6
- C) -30, -20
- D) -6, 6

Options C and D are invalid as roots because the points (-30, -20) are y-values, not x-values, and the roots are x-values where $p(x) = 0$.

Therefore, the roots are at $x = -6$ and $x = 6$.

Confirming Roots and Polynomial Equation

Using the Roots to Write the Polynomial

Given roots at $x = -6$ and $x = 6$, the polynomial can be expressed as:

$$p(x) = a(x + 6)(x - 6) = a(x^2 - 36)$$

Now, to determine the value of 'a', we use another point on the parabola.

Using the Point (0, -30)

Substitute $x = 0$, $p(0) = -30$:

$$-30 = a(0^2 - 36)$$
$$-30 = a(-36)$$
$$a = \frac{-30}{-36} = \frac{5}{6}$$

Thus, the quadratic polynomial is:

$$p(x) = \frac{5}{6}(x^2 - 36)$$

Verifying with Other Points

Let's check if the polynomial passes through the point (4, -20):

Calculate $p(4)$:

$$p(4) = \frac{5}{6}(4^2 - 36) = \frac{5}{6}(16 - 36) = \frac{5}{6}(-20) = -\frac{100}{6} = -16.\overline{6}$$

But the given point is (4, -20). Our calculation yields approximately -16.67, which does not match -20.

Similarly, check for $x=6$:

$$p(6) = \frac{5}{6}(36 - 36) = 0$$

This matches the point (6, 0).

Now, check at $x=4$:

- Calculated $p(4) \approx -16.67$
- Given $p(4) = -20$

The discrepancy suggests the polynomial is not a perfect quadratic passing through all points unless we consider measurement errors or more complex polynomial behavior.

However, the key is that the roots are at $x=-6$ and $x=6$, as these are the x-values where $p(x) = 0$, confirmed by the points.

Conclusion: The Zeroes of the Polynomial

Based on the above analysis:

- The polynomial passes through points with roots at x = -6 and x = 6.
- The options are:

- A) -6, 0
- B) 4, 6
- C) -30, -20
- D) -6, 6

- Only options A and D list x-values at -6 and 6. Since the roots are at x = -6 and x=6, options A and D are the candidates.

- Option A suggests roots at -6 and 0 (which is inconsistent with the data), whereas Option D suggests roots at -6 and 6, which aligns with the points (-6, 0) and (6, 0).

Therefore, the correct answer is D) -6, 6.

Implications and Applications in Mathematics

Understanding the roots of a quadratic polynomial is crucial in various mathematical contexts:

- Graphing Quadratics: Knowing the roots helps in sketching the parabola accurately.
- Factorization: Roots enable factorization into linear factors.
- Solving Equations: Roots are solutions to quadratic equations.
- Real-life Modeling: Roots can represent intersection points, maximum/minimum points, or equilibrium states.

Additional Insights for Learners

- Always verify roots by substituting into the polynomial.
- The roots are the x-intercepts of the graph; points where $p(x) = 0$.
- When given points on a graph, look for x-values where $p(x) = 0$ to identify roots.
- The symmetry of a parabola about its axis of symmetry can help locate roots.

Summary

- The points (-6, 0) and (6, 0) indicate roots at $x = -6$ and $x = 6$.
- The quadratic polynomial can be expressed as $p(x) = \frac{5}{6}(x^2 - 36)$.
- The zeroes of the polynomial are at $x = -6$ and $x = 6$, matching option D.

Final Verdict

The correct choice for the zeroes of the quadratic polynomial is D) -6, 6.

This analysis demonstrates how to interpret given points, identify roots, and formulate the quadratic polynomial, which are vital skills in algebra and calculus. Mastery of these concepts enhances problem-solving capabilities and deepens understanding of quadratic functions' behavior.

Remember: Always verify roots with substitution and consider the context of given points when analyzing quadratic graphs.

Frequently Asked Questions

What are the x-intercepts (roots) of the quadratic polynomial p(x) passing through (-6,0) and (6,0)?

The roots are $x = -6$ and $x = 6$, corresponding to options A and D.

Given the points (-6, 0), (0, -30), and (4, -20), how can we determine the quadratic polynomial p(x)?

We can set up a system of equations using these points and solve for the coefficients of $p(x) = ax^2 + bx + c$.

Why does the point (0, -30) suggest about the constant term c in p(x)?

Since $p(0) = -30$, the constant term $c = -30$.

How can the roots of the polynomial be confirmed using the given points?

By evaluating p(x) at $x = -6$ and $x = 6$, which yield $p(-6) = 0$ and $p(6) = 0$, confirming these as roots.

What is the significance of the point (4, -20) in determining the roots of p(x)?

It helps verify the shape of the parabola and supports that the roots are at $x = -6$ and $x = 6$, given the symmetry and the passing through these points.

Based on the points provided, which option correctly identifies the zeroes of the polynomial?

Option A) -6, 0 and Option D) -6, 6 are the zeroes; since $p(0) \neq 0$, the correct zeroes are -6 and 6, so answer is D.

Additional Resources

The graph of a quadratic polynomial p(x) passes through the points (-6,0), (0, -30), (4,-20) and (6,0). The zeroes of the polynomial are A) - 6,0 B) 4, 6 C) - 30,-20 D) - 6,6

Understanding the Problem: Quadratic Polynomials and Their Graphs

Quadratic polynomials are fundamental objects in algebra, characterized by their degree of two, and they take the general form:

$$p(x) = ax^2 + bx + c$$

where $a \neq 0$. The graph of a quadratic polynomial is a parabola that opens upward if $a > 0$ and downward if $a < 0$. One of the most notable features of a quadratic graph is its zeroes or roots—values of x where $p(x) = 0$.

In our case, we are given four points through which the quadratic passes:

- $(-6, 0)$
- $(0, -30)$
- $(4, -20)$
- $(6, 0)$

Our goal is to determine the zeroes of this polynomial, and specifically, identify which of the multiple-choice options correctly represent its roots.

Deciphering the Data: What Do the Points Tell Us?

Let's analyze the given points:

- $(-6, 0)$ and $(6, 0)$ are points where the parabola crosses the x-axis. These are potential roots or zeroes of the polynomial.

- $(0, -30)$ indicates the y-value when $x = 0$.

- $(4, -20)$ shows the y-value at $x = 4$.

- $(-6, 0)$ and $(6, 0)$ are particularly significant because they lie on the x-axis; this strongly suggests that these are roots of the polynomial.

However, the points $(0, -30)$ and $(4, -20)$ are not on the x-axis, so they are not roots, but they provide additional information about the shape and position of the parabola.

Identifying the Roots: Are -6 and 6 the Zeroes?

Given that the polynomial passes through \((-6, 0)\) and \((6, 0)\), these points satisfy \(p(-6) = 0 \) and \(p(6) = 0 \). This strongly indicates that \(-6\) and \(6\) are roots of the quadratic polynomial.

Since quadratic polynomials can have at most two roots, and these points are explicitly on the graph at zeroes, it's logical to infer that the roots are:

\[x = -6 \quad \text{and} \quad x = 6 \]

This aligns with options A and D in the multiple-choice list:

- A) \(-6, 0\)
- D) \(-6, 6\)

Option A suggests roots at \(-6\) and \(0\), but since \(p(0) = -30 \neq 0 \), the point \((0, -30)\) is not on the x-axis, eliminating option A. Therefore, the roots are more likely at \(-6\) and \(6\).

Confirming the Roots: Does the Data Support \(x = 4 \) as a Root?

Option B suggests roots at \(4 \) and \(6 \). Let's examine whether \(x=4 \) is a root.

- At \(x=4 \), \(p(4) = -20 \neq 0 \), so 4 is not a root.

This rules out option B.

Understanding the Other Options: Negative Values and Repeated Roots

Option C lists roots at \(-30\) and \(-20\), which are the y-values at specific points, not the x-intercepts. Since roots are x-values where \(p(x) = 0 \), this option does not make sense.

Option D suggests roots at \(-6\) and \(6\), which aligns with the points where the parabola crosses the x-axis, and is consistent with the initial observations.

Constructing the Quadratic: Verifying the Roots

Given that the points $(-6, 0)$ and $(6, 0)$ are roots, the quadratic polynomial can be expressed in factored form as:

$$p(x) = k (x + 6)(x - 6) = k (x^2 - 36)$$

where k is a constant coefficient to be determined.

To find k, use the point $(0, -30)$:

$$p(0) = k (0 + 6)(0 - 6) = k (6)(-6) = -36k$$

Given $p(0) = -30$:

$$-36k = -30$$

$$k = \frac{-30}{-36} = \frac{30}{36} = \frac{5}{6}$$

Thus, the quadratic polynomial is:

$$p(x) = \frac{5}{6} (x^2 - 36)$$

Verifying the Polynomial with the Other Points

Check whether the polynomial passes through the points $(4, -20)$:

$$p(4) = \frac{5}{6} (4^2 - 36) = \frac{5}{6} (16 - 36) = \frac{5}{6} (-20) = -\frac{100}{6} = -\frac{50}{3} \approx -16.67$$

But the given point is $(4, -20)$. Since $-16.67 \neq -20$, the polynomial doesn't exactly pass through $(4, -20)$ with this factorization.

Similarly, check $(6, 0)$:

$$p(6) = \frac{5}{6} (36 - 36) = 0$$

which matches the point $(6, 0)$.

And for \((0, -30)\):

\[p(0) = -30 \], as previously calculated, confirming our \(k\).

The discrepancy at \(x=4\) suggests that the quadratic polynomial passing through \((-6, 0)\), \((0, -30)\), and \((6, 0)\) approximates the points, but the points \((4, -20)\) and \((-6, 0)\) are likely approximate or perhaps part of a more complex polynomial.

However, since the question asks for the roots, and the points \((-6, 0)\) and \((6, 0)\) lie on the x-axis, the most logical conclusion is that the roots are at \(-6\) and \(6\).

Final Conclusion: Which Option Is Correct?

Based on the analysis:

- The parabola crosses the x-axis at \(-6\) and \(6\), confirming these as roots.
- The points with \(p(x) \neq 0 \) at \(x=0 \) and \(x=4 \) do not influence the roots directly.
- The polynomial's roots are at \(x = -6\) and \(x=6\).

Thus, the correct choice is D) \(-6, 6\).

Implications and Broader Context

Understanding the roots of quadratic polynomials from given points is a vital skill in algebra and calculus. It enables students and mathematicians to interpret the behavior of parabolas, analyze their symmetry, and apply this knowledge in real-world contexts like physics, engineering, and data modeling.

In this example, identifying the roots from the points where the polynomial crosses the x-axis simplifies the problem significantly. It underscores the importance of carefully examining data points and recognizing key features such as zeros, vertex, and symmetry.

Conclusion

Analyzing the given points reveals that the quadratic polynomial's zeros are at \(-6\) and \(6\). Multiple-choice option D correctly captures these roots, highlighting the importance of understanding the fundamental properties of quadratic functions and their graphs. Whether in academic settings or practical applications, mastering how to extract roots from data points is an essential step in the toolkit of anyone working with quadratic functions.

The Graph Of A Quadratic Polynomial P X Passes Through The Points 6 0 0 30 4 20 And 6 0 The Zeroes Of The Polynomial Are A 6 0 B 4 6 C 30 20 D 6 6

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