# The points A(18, -8) and B(-7, -8) are two vertices of an isosceles triangle ABC with AB = AC, if the altitude AN has the equation 4x + 3y = 48, find the coordinates of N and C.

**The points A(18, -8) and B(-7, -8) are two vertices of an isosceles triangle ABC with AB = AC, if the altitude AN has the equation 4x + 3y = 48, find the coordinates of N and C.**

## Understanding the Geometry of the Problem

When dealing with geometry problems involving triangles, especially isosceles triangles, it's essential to understand the relationships between different elements such as vertices, altitudes, and symmetry. In this problem, we are given two vertices of the triangle, A and B, and specific conditions about the altitude from vertex A to side BC.

The key details are:
- The vertices A(18, -8) and B(-7, -8).
- The triangle ABC is isosceles with AB = AC.
- The altitude from A, denoted as AN, has the equation 4x + 3y = 48.

Our goal is to find the coordinates of:
- The foot of the altitude N.
- The third vertex C.

To solve this, we need to understand the significance of the altitude, the properties of an isosceles triangle, and how the given line equation relates to the geometry.

## Step 1: Analyzing the Given Points and Triangle Properties

Let's consider the points:

- A(18, -8): a vertex of the triangle.
- B(-7, -8): another vertex, which lies horizontally aligned with A, since both share the y-coordinate -8.

Observation:
- The segment AB is horizontal because both points have y = -8.
- The length of AB can be calculated as:

$$
AB = |x_A - x_B| = |18 - (-7)| = 25
$$

Since the triangle is isosceles with AB = AC, side AC must also be of length 25.

Implications:
- The side AC is equal in length to AB.
- The point C lies such that AC is 25 units, and the triangle is symmetric with respect to the altitude from A.

## Step 2: Understanding the Altitude Line Equation

The altitude AN from A is perpendicular to BC and passes through point N, where N is the foot of the altitude on BC.

Given:

$$
4x + 3y = 48
$$

This is the equation of the line along which the altitude from A lies.

Key points:

- The line of the altitude passes through point N (on BC).
- Since N is on the line of the altitude, N must satisfy this equation.
- The altitude AN is perpendicular to BC, and the line BC contains point N.

Note:
- The foot of the altitude N isn't necessarily on the segment BC, but on its extension.
- The altitude AN passes through A(18, -8) and has the line equation 4x + 3y = 48.

## Step 3: Finding the Coordinates of N (Foot of the Altitude)

Since N lies on the altitude from A, and the line of the altitude is 4x + 3y = 48, point N must satisfy both the line equation and lie somewhere along the path from A perpendicular to BC.

Approach:

- The altitude passes through A(18, -8).
- To verify if A lies on the altitude line, check if A satisfies the line:

$$
4(18) + 3(-8) = 72 - 24 = 48
$$

Yes! The calculation confirms that A lies on the altitude line.

Conclusion:
- The foot of the altitude N is the point where the altitude line intersects the side BC, and since A is on this line, N coincides with A.

Therefore:

$$
\boxed{
N = (18, -8)
}
$$

This is a crucial insight: the foot of the altitude from A is at point A itself, indicating that the altitude from A falls directly on A, implying that the foot N is at A.

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Note: This suggests that the altitude from A is a degenerate case where N coincides with A. In the context of the problem, this indicates that the altitude from A is vertical or that N is A itself.

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# Step 4: Finding Coordinates of Point C

Since the triangle is isosceles with AB = AC, and we know:

- A(18, -8)
- B(-7, -8)
- AB length = 25

and that the altitude from A has the equation 4x + 3y = 48, which passes through A, we need to determine point C such that:

- AC = 25
- The triangle is symmetric with respect to the altitude from A.

Method:

- Find all points C such that:

$$
|AC| = 25
$$

- The set of all points C satisfying $|AC|=25$ lie on a circle centered at A with radius 25:

$$
(x - 18)^2 + (y + 8)^2 = 25^2 = 625
$$

- Also, point C lies on the line perpendicular to the altitude passing through N, which is at A.

Step 4a: Find the equation of the circle:

$$
(x - 18)^2 + (y + 8)^2 = 625
$$

Step 4b: Find the line(s) of symmetry; since the triangle is isosceles with AB = AC, the axis of symmetry passes through A and is perpendicular to BC.

Step 4c: Determine the possible positions of C:

- Since AB is horizontal, and the triangle is isosceles with AB = AC, C must be equidistant from A and B, with AC length 25.

- The point C lies on the circle centered at A with radius 25.

- The location of C depends on its position relative to the line AB.

Step 4d: Find specific coordinates for C:

Suppose C has coordinates $(x, y)$ satisfying the circle equation and the property that AC = 25.

From the circle:

$$
(x - 18)^2 + (y + 8)^2 = 625
$$

From the isosceles property and symmetry, C must be located such that the distances are equal, and the line of symmetry passes through A.

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Simplified Method to Find C:

Given that AB is horizontal, and the triangle is isosceles with AB = AC, then:

- The point C is located at a position symmetric to B with respect to the line of symmetry passing through A.

- Since B is at $(-7, -8)$, and A at $(18, -8)$, the midpoint of AB is at:

$$
\left( \frac{18 + (-7)}{2}, \frac{-8 + (-8)}{2} \right) = \left( \frac{11}{2}, -8 \right) = (5.5, -8)
$$

- The perpendicular bisector of AB is vertical (since AB is horizontal), at $x=5.5$.

- For C to satisfy AC = 25, and considering symmetry about this line, the possible positions for C are symmetric with respect to $x=5.5$.

- The distance from A to C:

$$
|AC| = 25
$$

- The x-coordinate of C:

$$
x_C = 2 \times 5.5 - x_B = 2 \times 5.5 - (-7) = 11 + 7 = 18
$$

- But this coincides with A's x-coordinate; thus, the symmetric point C is at:

$$
x_C = 18
$$

- To find y-coordinate, use the circle:

$$
(18 - 18)^2 + (y + 8)^2 = 625 \Rightarrow (0)^2 + (y + 8)^2 = 625
$$

\[
(y + 8)^2 = 625
\]

\[
y + 8 = \pm 25
\]

\[
y = -8 \pm 25
\]

So,

\[
\boxed{
y = 17 \quad \text{or} \quad y = -33
}
\]

Thus, the two possible positions for C are:

\[
C(18, 17) \quad \text{and} \quad C(18, -33)
\]

---

Final Coordinates:

C can be either:

\[
\boxed{
(18, 17) \quad \text{or} \quad (18, -33)
}
\]

Depending on the specific orientation of the triangle.

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# Summary of the Solution

- The foot of the altitude from A, N, coincides with A itself because A satisfies the line equation of the altitude.
- The third vertex C lies on the circle centered at A with radius 25, satisfying \((x - 18)^2 + (y + 8)^2 = 625\).
- Due to symmetry about the midpoint of AB, the possible positions for C are

at \((18, 17)\) and \((18, -33)\).

## Frequently Asked Questions

### Given points A(18, -8) and B(-7, -8), and an isosceles triangle ABC with AB = AC, how do you find the midpoint of AB?

**The midpoint of AB is found by averaging the x-coordinates and y-coordinates: ((18 + (-7))/2, (-8 + (-8))/2) = (5.5, -8).**

### What is the significance of the altitude AN with the equation 4x + 3y = 48 in triangle ABC?

**The altitude AN is perpendicular to BC and passes through vertex A, serving as a key line to locate the foot N and help determine point C.**

### How do you find the coordinates of point N, the foot of the altitude from A to BC, given the equation 4x + 3y = 48?

**Since N lies on the altitude line and the line passes through A(18, -8), substitute x=18 and**

y=-8 into the line equation: 4(18) + 3(-8) = 72 - 24 = 48, which satisfies the equation, so N is at (18, -8).

**Is point N the same as point A in this problem? Why or why not?**

Yes, because the foot of the altitude from A to BC lies on the line 4x + 3y = 48, and substituting A's coordinates satisfies this line, indicating N coincides with A.

**How do you determine the coordinates of point C in the isosceles triangle ABC?**

Since AB = AC and the triangle is isosceles with A at (18, -8), the point C must be equidistant from A as B is, and lie along the line perpendicular to BC passing through A.

**What method can be used to find the coordinates of C once N is known?**

Use the fact that AC = AB, find the distance AB, and then determine the possible locations for C based on symmetry about the altitude line, ensuring AC equals AB.

**What is the distance between points A(18, -8) and B(-7, -8)?**

**Distance AB = |18 - (-7)| = 25 units, since y-coordinates are the same (-8).**

**How do symmetry properties of an isosceles triangle help in locating point C?**

**The vertex A is equidistant from B and C, so C lies on the circle centered at A with radius equal to AB, and symmetric with respect to the altitude AN.**

**What are the possible coordinates of point C in this problem?**

**Point C can be found by reflecting B across the line passing through A along the perpendicular bisector of BC, resulting in C at (-32, -8).**

**Summarize the steps to find point C in this problem.**

**1. Find the distance AB = 25. 2. Determine the line of symmetry (perpendicular bisector of AB). 3. Use the symmetry and the altitude AN to locate C such that AC = AB and the triangle remains isosceles. 4. Calculate C's coordinates**

accordingly, which are (-32, -8).

Additional Resources

The points A(18, -8) and B(-7, -8) are two vertices of an isosceles triangle ABC with AB = AC, if the altitude AN has the equation 4x + 3y = 48, find the coordinates of N and C.

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Investigative Analysis: Determining Coordinates in an Isosceles Triangle with Given Conditions

In the realm of coordinate geometry, problems involving triangles with specific properties often serve as excellent gateways to understanding the relationships between geometric figures and algebraic equations. The problem at hand involves two fixed points, a condition on an altitude, and the task of finding the coordinates of a certain vertex. This investigation explores the problem thoroughly, uncovering the geometric relationships and algebraic steps necessary to determine the unknown points.

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## Framing the Problem

Given data:

- Points $A(18, -8)$ and $B(-7, -8)$.
- Triangle $ABC$ is isosceles with $AB = AC$.
- The altitude from $A$ onto side $BC$ is $AN$, with the equation $4x + 3y = 48$.

Goal: Find the coordinates of points $N$ (foot of the altitude from $A$ onto $BC$) and $C$.

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## Conceptual Foundations and Approach

Understanding the problem requires familiarity with several key geometric and algebraic concepts:

### Isosceles Triangle Properties

- In an isosceles triangle, two sides are equal, and the altitude from the apex (here, $A$) to the base ($BC$) bisects the base.
- The point $N$, being the foot of the altitude from $A$, lies on $BC$.

**Line Equations and Coordinate Geometry**

- **The line $4x + 3y = 48$ is the equation of the altitude $AN$.**
- **The line segment $BC$ must pass through $N$.**

**Strategy Overview**

**1. Identify the position of $N$: Since $N$ lies on the altitude line $4x + 3y = 48$, and $N$ is the foot of the perpendicular from $A$ onto $BC$, $N$ also lies somewhere along the segment $BC$.**

**2. Determine the nature of $BC$: Because $A$ is the vertex of an isosceles triangle with $AB = AC$, $N$ is the midpoint of $BC$ if the altitude from $A$ is also the median. But this depends on the triangle's configuration.**

**3. Use symmetry and algebra: Find possible locations of $C$ based on the given conditions and the equal sides.**

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**Step-by-Step Solution**

Step 1: Establish Coordinates and Known Elements

- $A(18, -8)$
- $B(-7, -8)$

Calculate $AB$:

$$
AB = \sqrt{(18 - (-7))^2 + (-8 - (-8))^2} = \sqrt{(25)^2 + 0^2} = 25
$$

Since $AB = AC$, then $AC = 25$. Therefore, point $C$ must lie at a distance 25 from $A$.

Step 2: Find the Foot of the Altitude $N$

The altitude from $A$ is along the line $4x + 3y = 48$. The foot $N$ is the point on this line such that $AN$ is perpendicular to $BC$.

Given that $AN$ is perpendicular to $BC$, the line $BC$ passes through $N$, and $N$ lies on the line $4x + 3y = 48$.

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**Step 3: Expressing $N$ in Coordinates**

**Let $N = (x_N, y_N)$. Since $N$ lies on $4x + 3y = 48$:**

$$
4x_N + 3y_N = 48
$$

**The point $A(18, -8)$, so the vector $AN$:**

$$
\vec{AN} = (x_N - 18, y_N + 8)
$$

**The line $AN$ is perpendicular to $BC$. Since $N$ is the foot of the perpendicular from $A$, the direction vector of $AN$ is perpendicular to the direction vector of $BC$.**

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**Step 4: Relationship Between $BC$ and $AN$**

**Suppose $B$ and $C$ are points on the same line $BC$, which passes through $N$. The line $BC$ must be perpendicular to $AN$, because in an altitude, the foot point $N$ ensures the line $AN$ is perpendicular to**

$BC$.

Thus, the direction vector of $BC$ is parallel to $AN^\perp$, which is perpendicular to $AN$.

---

Step 5: Find the Direction Vector of $AN$

The vector $AN$:

$$
\vec{AN} = (x_N - 18, y_N + 8)
$$

The slope of $AN$:

$$
m_{AN} = \frac{y_N + 8}{x_N - 18}
$$

The slope of $BC$:

$$
m_{BC} = -\frac{1}{m_{AN}}
$$

because $BC$ is perpendicular to $AN$.

---

Step 6: Express $N$ and $C$

Assuming $N$ is known, the line $BC$ passes through $N$ with slope $m_{BC}$. To find $C$, we need to find a point at a distance 25 from $A$, lying along the line through $N$ with slope $m_{BC}$.

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Deep Dive into Calculations

Step 7: Find $N$ Coordinates

To proceed, observe that the foot $N$ minimizes the distance from $A$ to the line $4x + 3y = 48$. The foot of the perpendicular from $A$ to this line is given by the formula:

$$
x_N = x_A - \frac{a(a x_A + b y_A + c)}{a^2 + b^2}
$$
$$
y_N = y_A - \frac{b(a x_A + b y_A + c)}{a^2 + b^2}
$$

where the line $ax + by + c = 0$.

Rewriting $4x + 3y = 48$ as:

$$
4x + 3y - 48 = 0
$$

So,

$$
a=4, \quad b=3, \quad c=-48
$$

Calculate:

$$
a x_A + b y_A + c = 4 \times 18 + 3 \times (-8) - 48 = 72 - 24 - 48 = 0
$$

Since this sum equals zero, the foot $N$ coincides with $A$. But this cannot be, because $A$ is at $(18, -8)$ and the line is $4x + 3y = 48$.

Wait: Let's verify this calculation carefully:

$$
a x_A + b y_A + c = 4 \times 18 + 3 \times (-8)

$$
- 48 = 72 - 24 - 48 = 0
$$

Indeed, it equals zero. This indicates that $A$ lies on the line $4x + 3y = 48$, meaning the altitude from $A$ onto $BC$ passes through $A$. Therefore, the foot $N$ is $A$ itself.

---

Key Observation: The Foot $N$ is at $A$

This simplifies the problem significantly:

- Since $A$ lies on the altitude line, the foot of the altitude $N$ is at $A(18, -8)$.

- The altitude $AN$ is from $A$ to $BC$, with $N=A$. So, the altitude is a point, and the line $BC$ passes through $A$.

- As $N = A$, the line $BC$ passes through $A$ and $N$.

---

Step 8: Find $C$ Given $AC = 25$

Since $AC = 25$, and $A(18, -8)$, $C$

must be at a distance 25 from $A$.

- The locus of points at distance 25 from $A$:

$$
(x - 18)^2 + (y + 8)^2 = 25^2 = 625
$$

- $C$ lies on this circle.

---

Step 9: Coordinates of $C$

Since $C$ lies on the circle centered at $A(18, -8)$, with radius 25, and on the line passing through $A$, the line $BC$ must pass through $A$. The

The Points A 18 8 And B 7 8 Are Two Vertices Of An Isosceles Triangle Abc With Ab Ac If The Altitude An Has The Equation 4x 3y 48 Find The Coordinates Of N And C

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## Coordinates Of N And C

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