The Work Done For A Particle Moves Once Counterclockwise About The Rectangle With The Vertices (0,1),(0,7),(3,1)

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Understanding the concept of work done by a force field along a path is fundamental in vector calculus and physics. When a particle moves along a specific trajectory, such as a rectangle, the work done by the force field depends on the nature of the field and the path taken. In this article, we delve into the detailed calculation of the work done by a vector field as a particle traverses once counterclockwise around the rectangle with vertices at (0,1), (0,7), and (3,1). We will explore the problem step by step, starting from the basic definitions to the application of Green's theorem, and finally interpret the physical significance of our results.

Understanding the Path and the Force Field

Vertices and Path Description

The rectangle in question is defined by three vertices: (0,1), (0,7), and (3,1). To visualize, these points form a closed figure when connected appropriately:

- Starting at point A: (0,1)
- Moving vertically up to B: (0,7)
- Then moving horizontally to C: (3,1)
- Finally returning back to A: (0,1)

The path is traversed once counterclockwise, which means the particle moves along the edges in the order: A → B → C → A.

Note: The points form a right-angled triangle, but because the problem states "rectangle," it's important to clarify that with the given vertices, the shape is a right triangle with a hypotenuse from B to C, and the path traces a rectangle-like figure when considering the proper closure. For simplicity, assume the path is the triangle formed by points (0,1), (0,7), and (3,1), traversed counterclockwise.

Force Field Description

To compute work, we need to specify the force field \(\vec{F}(x,y)\). Typically, the problem provides a force field such as:

\[
\vec{F}(x,y) = P(x,y) \, \mathbf{i} + Q(x,y) \, \mathbf{j}

\]

Suppose, for this problem, the force field is given as:

$$
\vec{F}(x,y) = (2x + y) \, \mathbf{i} + (x + 3y) \, \mathbf{j}
$$

This field is common in physics problems involving linear forces, and it allows for straightforward application of Green's theorem.

Important Note: If the force field is specified differently, the method remains similar; the main difference is in the integrand expressions.

Calculating the Work Done: Mathematical Background

Line Integral of a Vector Field

The work done by a force field $\vec{F}$ along a curve C is given by the line integral:

$$
W = \oint_C \vec{F} \cdot d\vec{r}
$$

Where:

- $d\vec{r} = dx \, \mathbf{i} + dy \, \mathbf{j}$
- The curve C is traversed once counterclockwise.

Expanding the dot product:

$$
W = \oint_C P(x,y) \, dx + Q(x,y) \, dy
$$

This integral sums the component of the force along the path over the entire loop.

Applying Green's Theorem

Green's theorem provides a powerful way to evaluate line integrals around closed curves:

$$
\oint_C P \, dx + Q \, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx \, dy
$$

Where:

- D is the region enclosed by C.

This theorem transforms the line integral into a double integral over the region D, often simplifying the calculations.

Note: For the given vertices, the region D is the triangle with vertices at (0,1), (0,7), and (3,1).

Step-by-Step Calculation of the Work Done

1. Defining the Boundary Path

The path C consists of three segments:

- Segment 1: from A (0,1) to B (0,7)
- Segment 2: from B (0,7) to C (3,1)
- Segment 3: from C (3,1) back to A (0,1)

Each segment will be parameterized for computation.

2. Parameterizing Each Segment

Segment 1: (0,1) to (0,7)

- $x = 0$
- $y = y$, y goes from 1 to 7

Differentials:

- $dx = 0$
- $dy = dy$

Line integral over this segment:

$$
W_1 = \int_{y=1}^{7} P(0, y) \cdot 0 + Q(0, y) \, dy
$$

Plug in $P(0,y) = 0 + y = y$, $Q(0,y) = 0 + 3 y = 3 y$:

$$
W_1 = \int_{1}^{7} 3 y \, dy = 3 \int_{1}^{7} y \, dy
$$

Calculating:

$$
W_1 = 3 \left[\frac{y^2}{2} \right]_1^7 = 3 \left(\frac{49}{2} - \frac{1}{2} \right) = 3 \times \frac{48}{2} = 3 \times 24 = 72
$$

Segment 2: (0,7) to (3,1)

- \(x\) varies from 0 to 3
- \(y\) varies from 7 to 1

Parameterization:

\[
x = x, \quad y = y(x) = 7 - 6 \times \frac{x}{3} = 7 - 2x
\]

Differentials:

\[
dx = dx, \quad dy = -2 dx
\]

Limits:

- \(x: 0 \to 3\)

Calculate the integral:

\[
W_2 = \int_{x=0}^{3} P(x, y(x)) \, dx + Q(x, y(x)) \, dy
\]

Express \(P\) and \(Q\):

\[
P(x, y) = 2x + y
\]
\[
Q(x, y) = x + 3 y
\]

Substitute \(y = 7 - 2x\):

\[
P(x) = 2x + (7 - 2x) = 2x + 7 - 2x = 7
\]
\[
Q(x) = x + 3 (7 - 2x) = x + 21 - 6x = -5x + 21
\]

Integral:

\[
W_2 = \int_0^3 P(x) \, dx + Q(x) \, dy
\]
\[
W_2 = \int_0^3 7 \, dx + \int_0^3 (-5x + 21) \, (-2 dx)
\]

Simplify:

\[

$$
W_2 = 7 \times 3 + \int_0^3 (-5x + 21) \times (-2) \, dx
$$

$$
W_2 = 21 + \int_0^3 (10x - 42) \, dx
$$

Calculate the integral:

$$
\int_0^3 10x \, dx = 5 x^2 \big|_0^3 = 5 \times 9 = 45
$$

$$
\int_0^3 -42 \, dx = -42 \times 3 = -126
$$

Adding:

$$
W_2 = 21 + (45 - 126) = 21 - 81 = -60
$$

Segment 3: (3,1) back to (0,1)

- $y = 1$ (constant)
- x from 3 to 0

Parameterization:

$$
x = x, \quad y=1
$$

$$
dx = dx, \quad dy=0
$$

Limits: $x: 3 \to 0$

Integral:

$$
W_3 = \int_{x=3}^{0} P(x,1) \, dx + Q(x,1) \times 0
$$

Calculate P and Q:

$$
P(x,1) = 2x + 1
$$

$$
Q(x,1) = x + 3 \times 1 = x + 3
$$

Since $dy=0$, the second term vanishes:

$$
W_
$$

Frequently Asked Questions

What is the significance of the path being a rectangle with vertices (0,1), (0,7), and (3,1)?

The rectangle defines the boundary along which the particle moves, providing a specific closed path in the coordinate plane that influences the work done by the force field around it.

How does the counterclockwise direction affect the calculation of work done by a force field?

Moving counterclockwise around a closed loop typically indicates a positive orientation, which affects the sign of the line integral used to compute work, often resulting in a positive value if the force field aligns with the path.

What methods can be used to calculate the work done by a force field along this rectangular path?

The work can be calculated using line integrals of the force field along the path, applying Green’s theorem if the field is conservative or if the integral simplifies through parameterization of the rectangle.

If the force field is conservative, what is the work done for the particle moving once around the rectangle?

If the force field is conservative, the work done over any closed path, including the rectangle, is zero because the potential difference around a closed loop is zero.

How does Green’s theorem simplify the calculation of work done around the rectangle?

Green’s theorem relates the line integral around a simple closed curve to a double integral over the region it encloses. This can simplify the work calculation by converting a line integral into an easier double integral when applicable.

What is the importance of the vertices' coordinates in determining the work done?

The vertices’ coordinates define the exact shape and size of the rectangle, which are essential in parameterizing the path and setting up the integrals for calculating work done by the force field.

Can symmetry of the force field impact the work done over the rectangular path?

Yes, symmetry in the force field can lead to simplifications or cancellations in the line integral, potentially resulting in zero work or other simplified results depending on the nature of the field.

Additional Resources

Work Done for a Particle Moving Once Counterclockwise Around a Rectangle with Vertices (0,1), (0,7), and (3,1)

Introduction to Work in Vector Fields

Understanding the work done by a force field as a particle moves along a path is a fundamental concept in vector calculus and physics. When a particle moves through a force field, the work done is a scalar quantity that encapsulates how much energy the force imparts to or extracts from the particle during the movement.

In this detailed exploration, we analyze the work done for a particle traversing once counterclockwise around a specific rectangle with vertices at points (0,1), (0,7), and (3,1). This involves dissecting the geometry, the nature of the force field (if specified), the parametrization of the path, and applying the principles of line integrals and Green's theorem to compute the work.

Understanding the Geometric Setup

Vertices of the Rectangle

The problem states the vertices as (0,1), (0,7), and (3,1). At first glance, these points suggest a triangle rather than a rectangle because only three vertices are specified. To clarify, consider the following:

- The vertices given are:
- A = (0, 1)
- B = (0, 7)
- C = (3, 1)

- These points form a triangle with vertices at A, B, and C.

Note: The description mentions a rectangle, but only three vertices are provided. Typically, a rectangle has four vertices. Given the points, it is more consistent to interpret the path as a triangle, unless there's a missing vertex or an error in the problem statement.

Assumption: The intended figure is a triangle with vertices at (0,1), (0,7), and (3,1). The path is to be traversed counterclockwise around this triangle.

Constructing the Triangle

- Vertices:
- $A = (0,1)$
- $B = (0,7)$
- $C = (3,1)$

- Sides:
- Segment AB: Vertical line from (0,1) to (0,7)
- Segment BC: From (0,7) to (3,1)
- Segment CA: From (3,1) back to (0,1)

- Orientation:
- Moving counterclockwise, the path would be:
1. Along AB from A to B
2. Along BC from B to C
3. Along CA from C back to A

Choosing the Force Field

Before computing the work, it's essential to specify the force or vector field acting on the particle. The problem statement does not explicitly define a force field, which is common in theoretical examinations where the field might be a standard one or implied.

Possible assumptions:

- The force field could be a given vector field such as:
- $\mathbf{F}(x,y) = P(x,y) \hat{i} + Q(x,y) \hat{j}$

- For a comprehensive analysis, a typical choice might be a conservative field like:
- $\mathbf{F}(x, y) = y \hat{i} + x \hat{j}$

- Or a more general field, such as:
- $\mathbf{F}(x, y) = (P, Q) = (x^2, y^2)$

For this review, we will proceed with a general approach and assume a vector field:

$$
\mathbf{F}(x, y) = P(x, y) \hat{i} + Q(x, y) \hat{j}
$$

where P and Q are continuous functions over the region enclosed by the triangle.

Line Integral and Work Calculation

Work as a Line Integral

The work W done by the force field $\mathbf{F}$ as the particle moves along a path C from point A to point B is given by the line integral:

$$
W = \int_{C} \mathbf{F} \cdot d\mathbf{r}
$$

where:

- $d\mathbf{r} = dx \hat{i} + dy \hat{j}$
- The integral sums the component of the force in the direction of movement along the path.

When the path C is closed, such as in this case moving once counterclockwise around the triangle, the work corresponds to the circulation of the field around C.

Parameterizing Each Segment

To evaluate the line integral, each side of the triangle must be parameterized:

1. Segment AB (from (0,1) to (0,7))

- Parameter: $t \in [0, 1]$
- $\mathbf{r}_{AB}(t) = (0, 1 + 6t)$
- $x = 0$, $y = 1 + 6t$
- $d\mathbf{r} = (0, 6) dt$

2. Segment BC (from (0,7) to (3,1))

- Parameter: $t \in [0, 1]$
- $\mathbf{r}_{BC}(t) = (3t, 7 - 6t)$
- $x = 3t$, $y = 7 - 6t$
- $d\mathbf{r} = (3, -6) dt$

3. Segment CA (from (3,1) back to (0,1))

- Parameter: $t \in [0, 1]$
- $\mathbf{r}_{CA}(t) = (3 - 3t, 1)$
- $x = 3 - 3t$, $y = 1$
- $d\mathbf{r} = (-3, 0) dt$

Applying Green's Theorem

For a closed curve C, Green's theorem offers an efficient way to compute the line integral:

$$
\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy
$$

where:

- D is the domain enclosed by C,
- $\mathbf{F} = (P, Q)$.

Advantages of Green's theorem:

- Converts a line integral around a closed curve into a double integral over the interior.
- Simplifies computations, especially for polygons like triangles.

Application conditions:

- $\mathbf{F}$ must be continuously differentiable within D,
- The curve C must be positively oriented (counterclockwise).

Implication for our problem:

- If the force field is conservative ($\nabla \times \mathbf{F} = 0$), the line integral around the closed loop is zero, implying no net work.

Case Study: Specific Force Field Example

To illustrate, assume a specific vector field:

$$
\mathbf{F}(x, y) = (y, -x)
$$

This is a rotational field with circulation around the origin.

- Compute the derivatives:

$$
\frac{\partial Q}{\partial x} = \frac{\partial (-x)}{\partial x} = -1
$$
$$
\frac{\partial P}{\partial y} = \frac{\partial y}{\partial y} = 1
$$

- The integrand for Green's theorem:

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -1 - 1 = -2
$$

- The double integral over the triangle:

$$
W = \iint_D -2 \, dx dy = -2 \times \text{Area}(D)
$$

$$
$$

- The area of triangle ABC:

$$
\text{Area} = \frac{1}{2} |x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)|
$$

Plugging in the points:

$$
\text{Area} = \frac{1}{2} |0(7 - 1) + 0(1 - 1) + 3(1 - 7)| = \frac{1}{2} |0 + 0 - 18| = 9
$$

- Therefore, the work:

$$
W = -2 \times 9 = -18
$$

This indicates that the net work done by the force field over the closed path is -18 units, implying a net energy transfer opposite to the direction of traversal (consistent with the rotation).

Implications and Insights

1. Conservative vs. Non-Conservative Fields

- If the force field is conservative, the net work over a

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