Use Newton's Method To Find Solutions Accurate To Within105for The Following Problems. A.ex+2x+2cosx6=0for1x2b.ln(x1)+cos(x1)=0for1.3x2c.2xcos2x(x2)2=0for2x3and3x4d.(x2)2lnx=0for1x2andex4e.ex3x2=0for0x1and3x5f.sinxex=0for0x13x4and6x7

Use Newton's Method To Find Solutions Accurate To Within 10−5 For The Following Problems. A. ex + 2x + 2cosx = 0 for 1 < x < 2; B. ln(x1) + cos(x1) = 0 for 1 < x1; C. 2x cos2x (x2) = 0 for 2 < x2 and 3 < x3; D. (x2) ln x = 0 for 1 < x2 and 4; E. ex 3x2 = 0 for 0 < x1 and 3 < x5; F. sin x ex = 0 for 0 < x1 < 3, 4, and 6 < x7

Introduction to Newton's Method and Its Importance

Newton's Method, also known as the Newton-Raphson method, is a powerful iterative technique used to find approximate solutions (roots) of real-valued functions. Its rapid convergence makes it highly valuable in solving nonlinear equations where analytical solutions are difficult or impossible to obtain. When seeking solutions with an accuracy of within 10−5, understanding how to properly implement Newton's Method is essential for precision and efficiency.

This article delves into applying Newton's Method to various complex problems, illustrating step-by-step procedures, convergence criteria, and practical considerations. The focus will be on the specific problems listed, demonstrating how to effectively approximate their solutions within the specified accuracy.

Fundamentals of Newton's Method

Basic Algorithm

Given a function f(x), Newton's Method iteratively refines an initial guess x0 using the formula:

$$
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
$$

\]

where:
- $f'(x)$ is the derivative of $f(x)$,
- x_n is the current approximation,
- x_{n+1} is the next approximation.

This process continues until the absolute difference between successive iterations $|x_{n+1} - x_n|$ is less than the desired tolerance (here, 10-5).

Convergence Criteria

Newton's Method converges quadratically near a simple root if:
- The initial guess is sufficiently close to the actual root,
- The derivative $f'(x)$ is not zero near the root.

To ensure the desired accuracy, iterations are performed until:

$$
|x_{n+1} - x_n| < 10^{-5}
$$

Applying Newton's Method to Specific Problems

Each problem involves different functions and domains. The approach includes:

- Defining the function $f(x)$,
- Computing its derivative $f'(x)$,
- Choosing an initial guess x_0,
- Iterating using Newton's formula,
- Checking for convergence within the specified tolerance.

Let's explore each problem in detail.

Problem A: Solving $e^x + 2x + 2\cos x = 0$ for $1 < x < 2$

Defining the Function and Its Derivative

- Function:

$$
f(x) = e^x + 2x + 2 \cos x
$$

- Derivative:

$$
f'(x) = e^x + 2 - 2 \sin x
$$

Initial Guess Selection

Analyzing the function at boundary points:

- At $x=1$:

$$
f(1) = e^1 + 2(1) + 2 \cos 1 \approx 2.718 + 2 + 2(0.5403) \approx 2.718 + 2 + 1.0806 = 5.7986
$$

- At $x=2$:

$$
f(2) = e^2 + 4 + 2 \cos 2 \approx 7.389 + 4 + 2(-0.4161) \approx 7.389 + 4 - 0.8322 = 10.556
$$

Since $f(x)$ is positive at both ends, but we need to find roots, check where $f(x)$ crosses zero. Let's evaluate at $x=1.5$:

$$
f(1.5) \approx e^{1.5} + 3 + 2 \times 0.0707 \approx 4.4817 + 3 + 0.1414 = 7.6231
$$

All are positive; perhaps the function does not cross zero in this interval. Let's check at $x=0.5$:

$$
f(0.5) \approx 1.6487 + 1 + 2 \times 0.8776 \approx 1.6487 + 1 + 1.7552 = 4.403
$$

Similarly positive. But considering the initial problem statement, it's possible the function crosses zero outside this interval, or perhaps there's a typo in the problem statement. Given the initial statement, we proceed with initial guesses in $[1,2]$, perhaps at $x=1.2$:

$$
f(1.2) \approx 3.3201 + 2.4 + 2 \times 0.3624 \approx 3.3201 + 2.4 + 0.7248 = 6.445
$$

No sign change detected; perhaps the root is outside the interval, or the function is always positive. For demonstration, assume a root near the lower end, say at $x=0.5$, and initial guess $x_0=0.5$.

Note: If the actual problem involves roots within $1 < x < 2$, further analysis or initial guesses in that interval are necessary. For the purpose of this demonstration, let's pick an initial guess $x_0=1.5$.

Iteration Process

Apply Newton's iteration:

$$
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
$$

Calculate $f(1.5)$:

$$
f(1.5) \approx 4.4817 + 3 + 0.1414 = 7.6231
$$

Calculate $f'(1.5)$:

$$
f'(1.5) = e^{1.5} + 2 - 2 \sin 1.5 \approx 4.4817 + 2 - 2 \times 0.9975 = 6.4817 - 1.995 = 4.4867
$$

Next approximation:

$$
x_1 = 1.5 - \frac{7.6231}{4.4867} \approx 1.5 - 1.699 \approx -0.199
$$

The new estimate is outside the initial interval, indicating the initial guess might not be optimal. Adjusting initial guesses or analyzing the function behavior further is necessary.

Practical Tip: Use graphing tools or software to identify approximate roots before applying Newton's Method.

Problem B: Solving $\ln(x) + \cos x = 0$ for $1 < x < 2$

Function and Derivative

- Function:

$$
f(x) = \ln x + \cos x
$$

- Derivative:

$$
f'(x) = \frac{1}{x} - \sin x
$$

Initial Guess

At $x=1$:

$$
f(1) = 0 + 1 = 1
$$

At $x=2$:

$$
f(2) = 0.6931 + (-0.4161) = 0.277
$$

Both positive; test at $x=1.5$:

$$
f(1.5) \approx 0.4055 + 0.0707 = 0.4762
$$

No sign change; perhaps the root is close to $x=1$. Let's check at $x=0.9$:

$$
f(0.9) \approx -0.1054 + 0.6216 = 0.5162
$$

Again positive. Since $f(x)$ remains positive, maybe the root exists where the function crosses zero near some value. Alternatively, if the problem's initial domain is strict, perhaps the solution is at a different interval.

Assuming the initial guess $x_0=1.2$:

Calculate $f(1.2)$:

\[
f(1.2) \approx

Frequently Asked Questions

How do you set up Newton's Method to solve the equation ex + 2x + 2cosx = 6 for x in the interval [1, 2]?

First, rewrite the equation as f(x) = ex + 2x + 2cosx - 6 = 0. Then, compute its derivative f'(x) = ex + 2 - 2sinx. Starting with an initial guess in [1, 2], apply Newton's iteration: $x_{n+1} = x_n - f(x_n)/f'(x_n)$, until the approximation is accurate within 10^{-5}.

What is the process to find a solution for ln(x1) + cos(x1) = 0 using Newton's Method?

Define f(x) = ln(x) + cos(x). Calculate f'(x) = 1/x - sin(x). Choose an initial guess $x_0 > 0$, then iteratively compute $x_{n+1} = x_n -$

f(x_n)/f'(x_n) until the solution converges to within 10^(-5).

How can Newton's Method be applied to solve 2x cos^2 x (x^2) = 0 for x in [2, 3]?

Rewrite as f(x) = 2x cos^2 x x^2 = 0. Simplify or analyze to find zeros, then compute the derivative f'(x). Starting within the interval, apply Newton's iteration: x_{n+1} = x_n - f(x_n)/f'(x_n), stopping once the difference is less than 10^(-5).

Describe the steps to solve (x^2) ln x = 0 for x in [1, 4] using Newton's Method.

Set f(x) = x^2 ln x. Its derivative is f'(x) = 2x ln x + x. Choose an initial guess in [1, 4], then apply x_{n+1} = x_n - f(x_n)/f'(x_n) iteratively until the desired accuracy of 10^(-5) is achieved.

How do you find solutions to the equation e^{x^3} = 0 in [0, 1] using Newton's Method?

Note that e^{x^3} is never zero, so the equation has no real solutions. Therefore, Newton's Method cannot find solutions in this case; the method confirms no solutions exist in the interval.

Additional Resources

Use Newton's Method To Find Solutions Accurate To Within 10^{-5} for the Following Problems

---

Introduction

Numerical methods are pivotal tools in modern scientific computation, especially when analytical solutions are either challenging or impossible to obtain. Among these, Newton's Method (also known as the Newton-Raphson method) stands out for its simplicity, efficiency, and quadratic convergence rate near roots. This technique iteratively refines an initial guess to approximate solutions of real-valued functions with high precision.

In this comprehensive review, we explore the application of Newton's Method to a series of nonlinear equations, aiming to find solutions accurate to within 10^{-5} (i.e., five decimal places). These problems are representative of typical challenges faced in numerical analysis, involving diverse function types such as polynomials, logarithmic, trigonometric, and composite functions.

Fundamentals of Newton's Method

Newton's Method is based on the idea of linear approximation of a function $f(x)$ around a current estimate x_n:

\[

$$
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
$$

where:

- $f(x)$ is the function whose root is sought.
- $f'(x)$ is the derivative of $f(x)$.
- x_n is the current approximation.
- x_{n+1} is the next approximation.

Convergence Criterion:
The process continues iteratively until the absolute difference between successive approximations satisfies:

$$
|x_{n+1} - x_n| < 10^{-5}
$$

or equivalently, the residual $f(x_{n+1})$ is sufficiently close to zero.

Prerequisites for Success:

- A good initial guess x_0.
- The function $f(x)$ must be differentiable near the root.
- The derivative $f'(x)$ should not be zero at the root.

Problem 1: $e^{x} + 2x + 2 \cos x - 6 = 0$ for $x \in [1, 2]$

Analysis and Approach

This nonlinear equation combines exponential, linear, and trigonometric components. The goal is to find a root within the interval where the function changes sign.

Step 1: Identify an initial guess.

Evaluate the function at the interval endpoints:

- $f(1) = e^{1} + 2(1) + 2 \cos 1 - 6 \approx 2.718 + 2 + 2 \times 0.5403 - 6 \approx 2.718 + 2 + 1.0806 - 6 \approx -0.2014$

- $f(2) = e^{2} + 4 + 2 \cos 2 - 6 \approx 7.389 + 4 + 2 \times -0.4161 - 6 \approx 7.389 + 4 - 0.8322 - 6 \approx 4.5568$

Since $f(1) < 0$ and $f(2) > 0$, the root lies between 1 and 2. A natural initial guess is $x_0 = 1.5$.

Step 2: Derivative calculation:

$$
f'(x) = e^{x} + 2 - 2 \sin x
$$

Step 3: Iterative process:

Applying Newton's Method:

$$
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
$$

Iterate until $|x_{n+1} - x_n| < 10^{-5}$.

Practical Implementation:

- Use computational tools (e.g., Python, MATLAB) to automate convergence.
- The process converges rapidly owing to quadratic convergence.

Expected Result: Approximately $x \approx 1.35$, with five decimal place accuracy.

Problem 2: $\ln(x_1) + \cos(x_1) = 0$ for $x \in [1, 3]$

Analysis and Approach

This equation involves a logarithmic and a trigonometric function. To apply Newton's Method:

Initial guess:

- At $x=1$, $f(1) = 0 + \cos 1 \approx 0.5403$

- At $x=3$, $f(3) = \ln 3 + \cos 3 \approx 1.0986 - 0.9899 \approx 0.1087$

Since the function is positive at both ends, but we seek where it crosses zero, check at $x=2$:

$$
f(2) = \ln 2 + \cos 2 \approx 0.6931 - 0.4161 \approx 0.277
$$

All are positive, indicating the root is around where the function transitions from positive to negative.

Check at $x \approx 0.5$:

$$
f(0.5) = \ln 0.5 + \cos 0.5 \approx -0.6931 + 0.8776 \approx 0.1845
$$

No sign change observed; hence, the root is likely near $x \approx 0.3$:

- At $x=0.3$:

$$
f(0.3) = \ln 0.3 + \cos 0.3 \approx -1.2039 + 0.9553 \approx -0.2486
$$

Since $f(0.5)>0$ and $f(0.3)<0$, the root is between 0.3 and 0.5. Using $x_0=0.4$ as initial guess.

Step 2: Derivative:

$$
f'(x) = \frac{1}{x} - \sin x
$$

Step 3: Iterations:

Implement Newton's method iteratively to refine the root estimate until the desired accuracy is achieved.

Problem 3: $2x \cos^2 x (x^2) = 0$ for $x \in [2, 3]$

Simplification and Analysis

Rearranged:

$$
f(x) = 2x \cos^2 x \times x^2 = 0
$$

Since the product is zero, roots occur when any factor is zero:

- $x=0$ (outside the interval).

- $\cos x=0$, i.e., $x = \frac{\pi}{2} + n\pi$.

Within $[2, 3]$, $\pi/2 \approx 1.57$, and 3 is close to 2.36. The next root:

$$
x = \frac{\pi}{2} + \pi \approx 1.57 + 3.14 \approx 4.71
$$

which is outside $[2,3]$. So, within the interval, no zeros from $\cos x=0$.

The other factor:

- $x=0$: outside interval.

- $x^2=0$: at $x=0$, outside interval.

Thus, no roots in the interval, unless considering the trivial roots at the endpoints, which are not zeros.

Conclusion: No solutions to find via Newton's method here within $[2,3]$.

Problem 4: $(x^2) \ln x = 0$ for $x \in [1, 4]$

Analysis and Solution

Set the function:

$$
f(x) = x^2 \ln x
$$

Roots occur when:

- $x=1$: since $\ln 1=0$, $f(1)=0$.
- $x=0$: outside the interval.

Within $[1, 4]$, $x=1$ is a root.

Further analyze:

- For $x>1$, $\ln x>0$, so $f(x)>0$.
- At $x=4$:

$$
f(4)=16 \times \ln 4 \approx 16 \times 1.3863 \approx 22.18
$$

No other roots in $[1, 4]$. The only root is at $x=1$.

Therefore, the solution is known analytically, but for practice, we can verify the root and refine if necessary.

Problem 5: $e^{x} - 3x^{2} = 0$ for $x \in [0, 3]$

Analysis and Approach

Define:

$$
f(x) = e^{x} - 3x^{2}
$$

Evaluate at interval bounds:

- At $x=0$:

$$
f(0)=1 - 0=1>0
$$

- At $x=3$:

$$
f(3)=e^{3} - 3 \times 9 \approx 20.0855 - 27 = -6.9145<0
$$

Sign change indicates a root between 0 and 3.

Initial

Use Newton S Method To Find Solutions Accurate To

Within105for The Following Problems A Ex 2x 2cosx6 0for1x2b Ln X1 Cos X1 0for1 3x2c 2xcos2x X2 2 0for2x3and3x4d X2 2lnx 0for1x2andex4e Ex3x2 0for0x1and3x5f Sinxex 0for0x13x4and6x7

Use Newton S Method To Find Solutions Accurate To Within105for The Following Problems A Ex 2x 2cosx6 0for1x2b Ln X1 Cos X1 0for1 3x2c 2xcos2x X2 2 0for2x3and3x4d X2 2lnx 0for1x2andex4e Ex3x2 0for0x1and3x5f Sinxex 0for0x13x4and6x7

Related Articles

- Completa Las Siguientes Oraciones (Usar "a")1 (tu)_____________(encantar) Las Pelculas2 (El)______________(aburrir)
- Thomas Jefferson based his argument for unalienable rights on the work of which person? 17 19 16 John Locke Jean-Jacques Rousseau 201 Baron de Montesquieu Voltaire
- When Defining And Initializing A Final Variable Within A Class, It Usually Makes Sense To Make The Variable:Question

Back to Home