What Is The Simplest Sum-of-products For The Following Function? F(x1,x2,x3)=m(3,4,6,7) X2x3+x1x3 X2x3+x1x3x2x3+x1x3x2x3+x1x3

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Understanding and simplifying Boolean functions is a fundamental skill in digital logic design. The process involves reducing complex logical expressions into their simplest sum-of-products (SOP) form, which minimizes the number of logic gates required for implementation. In this article, we will analyze the given function:

F(x1, x2, x3) = m(3,4,6,7) + X2x3 + X1x3 + X1x2x3 + X1x2x3 + X1x3

and determine its simplest SOP expression. To do this systematically, we'll explore the function's minterms, utilize Karnaugh maps, and apply Boolean algebra rules.

Understanding the Given Function

Before simplifying, it's essential to interpret the given function correctly and identify its components.

Minterm Notation: m(3,4,6,7)

The notation m(3,4,6,7) indicates that the function F is true (i.e., equals 1) for the minterms numbered 3, 4, 6, and 7 in the 3-variable truth table. Let's convert these minterm indices to binary:

Minterm	Binary (x1 x2 x3)	Corresponding Variables
3	011	$x_1=0, x_2=1, x_3=1$
4	100	$x_1=1, x_2=0, x_3=0$
6	110	$x_1=1, x_2=1, x_3=0$
7	111	$x_1=1, x_2=1, x_3=1$

Based on this, the minterm expression can be written as:

F_minterms = □m(3,4,6,7) = □m(011,100,110,111)

which translates to the sum of these minterms.

Additional Terms in the Expression

The expression also contains terms like:

- x2x3
- x1x3
- x1x2x3

Note that these appear to be product terms (AND operations). However, the original expression seems to have some formatting issues, but assuming standard notation, the full function appears to be:

F = m(3,4,6,7) + x2x3 + x1x3 + x1x2x3 + x1x2x3 + x1x3

Note that the term x1x2x3 appears twice, which is redundant. Also, the presence of multiple terms involving x2x3 and x1x3 suggests overlapping minterms.

Interpreting the Function Components

To simplify, let's express the function step by step.

Expressing Minterms

The minterms m(3,4,6,7) in sum-of-products form:

- m(3): x1' x2 x3
- m(4): x1 x2' x3'
- m(6): x1 x2 x3'
- m(7): x1 x2 x3

Thus, the minterm sum is:

F_minterms = x1' x2 x3 + x1 x2' x3' + x1 x2 x3' + x1 x2 x3

Additional Product Terms

The other terms are:

- x2 x3
- x1 x3
- x1 x2 x3

Note that x2 x3 is true when x2=1 and x3=1, regardless of x1.

x1 x3 is true when x1=1 and x3=1, regardless of x2.

x1 x2 x3 is true when all three are 1.

Constructing the Karnaugh Map

A Karnaugh map (K-map) helps visualize the function and identify overlaps for simplification.

Setting Up the K-Map

For three variables, the K-map has 8 cells, labeled as follows:

	x2x3=00	x2x3=01	x2x3=11	x2x3=10
x1=0	0	1	3	2
x1=1	4	5	7	6

Mapping minterms:

- 3 (0,1,1): x1=0, x2=1, x3=1 cell (x1=0, x2x3=11)
- 4 (1,0,0): x1=1, x2=0, x3=0 cell (x1=1, x2x3=00)
- 6 (1,1,0): x1=1, x2=1, x3=0 cell (x1=1, x2x3=10)
- 7 (1,1,1): x1=1, x2=1, x3=1 cell (x1=1, x2x3=11)

Now, mark the minterms:

	00	01	11	10
0	0	0	1	0
1	1	0	1	1

Where "1" indicates the minterm or included term.

Simplification Process

Step 1: Combine Minterms

Group the 1s in the K-map to find prime implicants:

- Group 1: cells 3, 7, 6, 4 (covering all minterms), but since only minterms 3,4,6,7 are true, focus on minimal groups.

- Groupings:

- Cell 3 (x1=0, x2=1, x3=1) and cell 7 (x1=1, x2=1, x3=1): Both share x2=1, x3=1, differing in x1.

- Simplifies to x2 x3

- Cell 4 (x1=1, x2=0, x3=0) and cell 6 (x1=1, x2=1, x3=0): Both share x1=1, x3=0, differing in x2.

- Simplifies to x1 x3'

- Also, minterm 3 is covered by x2 x3, since x2=1, x3=1.

- Minterm 4 is covered by x1 x3',

- Minterm 6 is covered by x1 x3'

- Minterm 7 is covered by x1 x2 x3, but note that it's already covered by x2 x3 (since x2=1, x3=1).

Step 2: Write Prime Implicants

From the above, prime implicants are:

- x2 x3 (covers minterms 3 and 7)

- x1 x3' (covers minterms 4 and 6)

- x1 x2 x3 (covers minterm 7, but it's already covered by x2 x3, so possibly redundant)

Now, check for redundancies.

Step 3: Final Simplified SOP Expression

Considering the prime implicants:

- x2 x3 (covers minterms 3, 7)

- x1 x3' (covers minterms 4, 6)

- The minterm 3 is covered by x2 x3.

- Minterm 4 is covered by x1 x3'.

- Minterm 6 is covered by x1 x3'.

- Minterm 7 is covered by x2 x3.

Thus, the minimal SOP expression is:

F = x2 x3 + x1 x3' + x1 x2 x3

Note that x1 x2 x3 is redundant because it's covered by x2 x3 (since x2=1, x3=1, x1=1). So, including x2 x3 covers minterms 3 and 7, which are also covered by x2 x3.

Therefore, the simplest SOP form is:

F = x2 x3 + x1 x3' + x1 x2 x3

But since x1 x2 x3 is covered by x2 x3, it can be omitted:

F = x2 x3 + x1 x3'

Final Simplified Expression

Combining all the above steps, the most simplified sum-of-products form of the given function is:

F(x1, x2, x3) = x2 x3 + x1 x3'

This expression is minimal

Frequently Asked Questions

What is the original Boolean function given in the problem?

The function is F(x1, x2, x3) = m(3, 4, 6, 7) + x2x3 + x1x3 + x1x2x3 + x1x3x2x3 + x1x3.

How can the minterms m(3, 4, 6, 7) be expressed in Boolean terms?

Minterms 3, 4, 6, and 7 correspond to binary inputs: 3=011, 4=100, 6=110, 7=111, which translate to specific minterm expressions involving x1, x2, and x3.

What simplification technique should be used to find the simplest sum-of-products form?

Karnaugh Map (K-map) simplification is typically used to group minterms and simplify the Boolean expression efficiently.

What is the simplified sum-of-products form of the function?

The simplest SOP form of the function is F = x2 + x1x3.

Why is the form F = x2 + x1x3 considered the simplest SOP form?

Because it covers all the original minterms with the minimal number of product terms and literals, ensuring optimal simplicity and implementation efficiency.

Additional Resources

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Introduction

In digital logic design, simplifying Boolean functions is a fundamental task that directly impacts the efficiency, cost, and performance of digital circuits. Among the methods used for simplification, the sum-of-products (SOP) form is one of the most straightforward and widely adopted. The SOP form expresses a Boolean function as a logical OR (sum) of multiple AND (product) terms, where each term is a conjunction of literals (variables or their complements).

This article delves into a specific Boolean function:

$$ F(x_1, x_2, x_3) = m(3, 4, 6, 7) + x_2 x_3 + x_1 x_3 + x_1 x_3 x_2 x_3 + x_1 x_3 x_2 x_3 + x_1 x_3 $$

The goal is to determine the simplest sum-of-products expression for this function. To do this, we will analyze the function comprehensively, understand the minterm notation, and systematically derive the minimal SOP form.

Understanding the Function

The Minterm Notation: $m(3, 4, 6, 7)$

In Boolean algebra, the notation $m(i)$ denotes the minterm with decimal index i. For three variables x_1, x_2, x_3, minterms are represented as binary combinations:

Decimal	Binary (x1 x2 x3)	Minterm Expression
0	000	$\overline{x}_1 \overline{x}_2 \overline{x}_3$
1	001	$\overline{x}_1 \overline{x}_2 x_3$
2	010	$\overline{x}_1 x_2 \overline{x}_3$
3	011	$\overline{x}_1 x_2 x_3$
4	100	$x_1 \overline{x}_2 \overline{x}_3$
5	101	$x_1 \overline{x}_2 x_3$
6	110	$x_1 x_2 \overline{x}_3$
7	111	$x_1 x_2 x_3$

Thus,

$$ m(3, 4, 6, 7) = $$

$$ \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 $$

Dissecting the Boolean Function

The full expression given is:

$$ F(x_1, x_2, x_3) = m(3,4,6,7) + x_2 x_3 + x_1 x_3 + x_1 x_3 x_2 x_3 + x_1 x_3 x_2 x_3 + x_1 x_3 $$

Notice that some terms are repeated or redundant, and some are expressions with overlapping terms. Let's analyze each component:

- Minterm sum: $m(3,4,6,7)$ comprises minterms where the function is 1.
- Additional terms:
- $x_2 x_3$
- $x_1 x_3$
- $x_1 x_3 x_2 x_3$ (which simplifies to $x_1 x_2 x_3$)
- $x_1 x_3 x_2 x_3$ (duplicate of the above)
- $x_1 x_3$ (appears again)

Given the redundancy, it appears there are multiple repetitions and possible simplifications.

Step-by-Step Simplification Process

1. Eliminating Redundancies

Observe the terms:

- $x_1 x_3 x_2 x_3$ simplifies to $x_1 x_2 x_3$ because $x_3 x_3 = x_3$.
- The term $x_1 x_3 x_2 x_3$ appears twice, which is redundant.
- The repeated $x_1 x_3$ can be combined.

So, the expression reduces to:

$$F = m(3,4,6,7) + x_2 x_3 + x_1 x_3 + x_1 x_2 x_3$$

Note: $m(3,4,6,7)$ already covers some minterms. Let's explicitly write out the minterms:

$$m(3,4,6,7) = \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3$$

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2. Constructing the Karnaugh Map (K-Map)

To visualize the function and find the minimal SOP form, constructing a 3-variable K-Map is ideal.

Variables:

- Rows: x_1
- 0: $\overline{x}_1$
- 1: x_1
- Columns: $x_2 x_3$
- 00: $\overline{x}_2 \overline{x}_3$
- 01: $\overline{x}_2 x_3$
- 11: $x_2 x_3$
- 10: $x_2 \overline{x}_3$

Populate the K-Map:

	00 ($\overline{x}_2 \overline{x}_3$)	01 ($\overline{x}_2 x_3$)	11 ($x_2 x_3$)	10 ($x_2 \overline{x}_3$)
$x_1=0$	$\overline{x}_1 \overline{x}_2 \overline{x}_3$	$\overline{x}_1 \overline{x}_2 x_3$	$\overline{x}_1 x_2 x_3$	$\overline{x}_1 x_2 \overline{x}_3$
$x_1=1$	$x_1 \overline{x}_2 \overline{x}_3$	$x_1 \overline{x}_2 x_3$	$x_1 x_2 x_3$	$x_1 x_2 \overline{x}_3$

Now, mark the minterms from $m(3,4,6,7)$:

- 3: $\overline{x}_1 x_2 x_3$ → row 0, col 11
- 4: $x_1 \overline{x}_2 \overline{x}_3$ → row 1, col 00
- 6: $x_1 x_2 \overline{x}_3$ → row 1, col 10
- 7: $x_1 x_2 x_3$ → row 1, col 11

Now, include the other terms:

- $x_2 x_3$: covers all entries where $x_2=1, x_3=1$ → columns 11, regardless of x_1:
- (0,11), (1,11)
- $x_1 x_3$: covers all where $x_1=1, x_3=1$:
- (1,01), (1,11)
- $x_1 x_2 x_3$: already included in the minterm $m(7)$.

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3. Identifying Prime Implicants

From the K-Map, identify groups:

- Group A: All cells with \(x_2=1, x_3=1 \)

Cells: (0,11) and (1,11)

- Group B: All cells with \(x_1=1, x_3=1 \)

Cells: (1,01) and (1,11)

- Group C: Minterm 4 alone: (1,00)

- Group D: Minterm 6 alone: (1,10)

- Group E: Minterm 3 alone: (0,11)

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